

Solutions to Practice Test 1

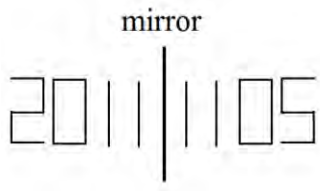
1. Solution: 2011
The sum of the digits of 2001, 2004, 2007, and 2013 is divisible by 3. So the answer is 2011. You can check all the prime numbers less than 44 to verify if you like.

2. Solution:
By the definition, an obtuse angle is one which is more than 90° but less than 180°. So we have only one obtuse angle: 126°.

3. Solution: 67.

The numbers should be divisible by 2 × 3 × 5 = 30. $\left\lfloor \frac{2011}{30} \right\rfloor = 67$

4. Solution: 1105.
The new figure is symmetrical with the original one about the mirror as shown.



5. Solution: 204.

The number of three-digit decreasing positive integers: $\binom{10}{3} = 120$.

The number of three-digit increasing positive integers: $\binom{9}{3} = 84$.

The answer is 120 + 84 = 204.

6. Solution: 503.
We see the pattern of the last digit: 7, 9, 3, 1. Every 4 number has one. 2011= 502 ×4 +3)

7. Solution: 4.
$\frac{6x+3}{2x-1} = \frac{6x-3+6}{2x-1} = 2 + \frac{6}{2x-1}$.

So 6 must be divisible by $2x - 1$. Since 6 has 4 factors, we know that $2x - 1$ can be any one of them.

8. Solution: 4.
When we remove the cube A, we take 3 faces off. At the same time we create take 3 faces. So change in surface area is 0.

9. Solution: 254.
We see that the pattern is repeated every 8 numbers. $2011 = 1 + (n - 1) \times 2$.
$n = 1006$
$1006 = 125 \times 8 + 6$.
So the number 2011 is in the $125 \times 2 + 1 = 251^{st}$ row and third column. The answer is $251 + 3 = 254$.

10. Solution: 99.
Method 1:

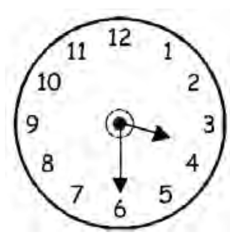
3 8 15 24
5 7 9
2 2

By Newton's Formula, we have

$$a_n = A\binom{n-1}{0} + B\binom{n-1}{1} + C\binom{n-1}{2} + \cdots\cdots + K\binom{n-1}{m} =$$
$$3\binom{8}{0} + 5\binom{8}{1} + 2\binom{8}{2} = 3 + 40 + 56 = 99.$$

Method 2:
We observe the pattern that the number of triangles in nth figure is $n^2 - 1$.
So the answer is $10^2 - 1 = 99$.

11. Solution: 1.
We see that the pattern is repeated for every 6 numbers.
$2011 = 335 \times 6 + 1$. So the answer is 1.



12. Solution: 75°.
The angle is 30° + 30° + 15° = 75°.

13. Solution: 3.
The prime numbers are 2, 3, 5, 7, 11, 13, 17, and 19. $3a + 2b = 3a + 3b - b = 3(a + b) - b$
We see that $a \neq b$ (if $a = b$, $3a + 2b$ will not be a prime number) and $a + b$ is at least 5 and at most 8.
So $19 = 3 \times (3 + 5) - 5 = 3 \times (5 + 2) - 2$. $13 = 3 \times (2 + 3) - 2$.
We have 3 pairs.

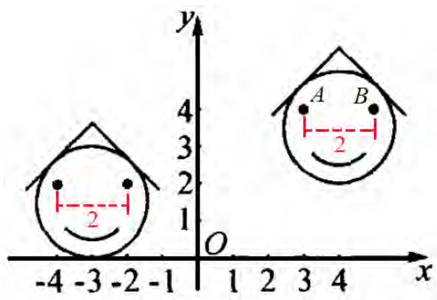
14. Solution: -1.
We have $\frac{2x+y^3}{2} = \frac{2\times 3+(-2)^3}{2} == -1$

15. Solution: 4/9.
$4 = 2 + 1 + 1$.
The number of ways to distribute the 4 balls in such a way is
$\binom{4}{2}\binom{2}{1}\binom{1}{1}\times\frac{3!}{2!} = 36$
m distinct balls are put into n labeled boxes and the number of balls in each box is not limited. The number of ways to do so is $N = n^m = 3^4$.

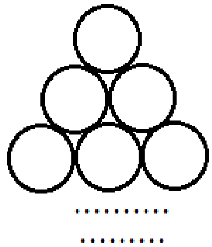
The probability is $P = \frac{36}{3^4} = \frac{4}{9}$.

16. Solution: 19/125.
We have the following cases:
$9 = 1 + 3 + 5 = 1 + 4 + 4 = 2 + 2 + 5 = 2 + 3 + 4 = 3 + 3 + 3$.
$P = \frac{6+3+3+6+1}{5^3} = \frac{19}{125}$.

17. Solution: 9.
The distance from A to B is 2. So the coordinate of B is (4, 5). The sum is 4 + 5 = 9.

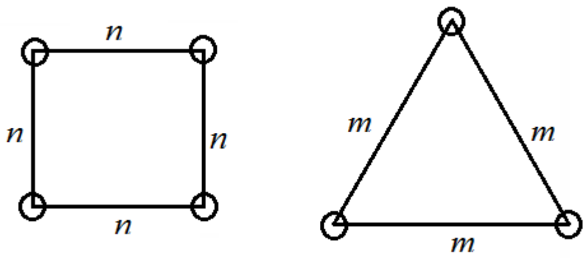


18. Solution: 36.
Method 1:
When the balls packed to form an equilateral triangle shape, the first row has one bal, second row has 2 balls,..



We see that $\frac{(1+7)7}{2}=28$, $\frac{(1+8)8}{2}=36$, $\frac{(1+9)9}{2}=45$.
So the number of small balls is 36 which is also a square number.

Method 2:
Let n be the number of balls in each row of the square without counting the balls in each corner.
Let m be the number of balls in each row of the triangle without counting the balls in each corner.



We have $4 + 4n = 3 + 3m \Rightarrow 4(1 + n) = 3(1 + m)$. So the number of small balls is a multiple of 12. Since $m > n \geq 1$. $1 + n$ is a multiple of 3 and $n < 9$. We checked and n = 8 works. So the number of small balls is 36.

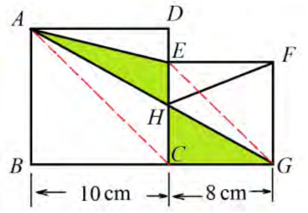
19. Solution: 6.

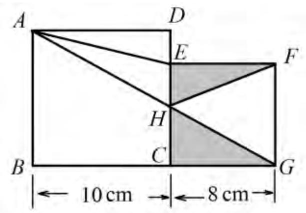
We can have $-4x^2$, -1, $4x$, $-4x$, $4x^4$, and $\frac{1}{16x^2}$.

20. Solution: 32π.
Let the radius of the circle be r.
The sides of the rectangle $BCEF$ are $4r$ and $4\pi + 4 - 2r$. When it forms the cylinder, one side is the height and another side is the circumference of the circle. We see that only $4\pi + 4 - 2r$ can be the circumference. So we have $4\pi + 4 - 2r = 2\pi r$. Solving we get $r = 2$.

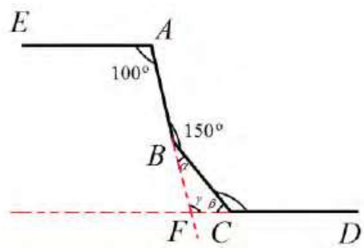
The volume is $\pi r^2 h = \pi r^2 (4r) = 4\pi \times 2^3 = 32\pi$

21. Solution: 32.
Connect *AC* and *EG*. We see that triangles *AEH* and *CHG* have the same area. So our problem is now finding the sum of the areas of triangles *EFH* and *CHG*. We see that the sum of the areas is half of the area of the 8 by 8 square. So the answer is 32.





22. Solution: 130°.
Extend *AB* and *DC* to meet at *F*. We see that γ = ∠*A*= 100°.
Since 150° = γ + β, so β = 150° – γ = 150° – 100° = 50°.
Thus ∠*C* 180° – 50° = 130°.



23. Solution: 2.
The equation of the line L can be written as $\frac{1}{a} + \frac{3}{b} = 1 \quad \Rightarrow \quad 3a + b = ab \quad \Rightarrow$

$$(a-1)(b-3) = 3$$

$$\begin{cases} a-1=1 & \Rightarrow & a=2 \\ b-3=3 & \Rightarrow & b=6 \end{cases}$$

$$\begin{cases} a-1=3 & \Rightarrow & a=4 \\ b-3=1 & \Rightarrow & b=4. \end{cases}$$

24. Solution: 85.
$\frac{20 \times 85 + 1 \times 85}{20+1} = \frac{85 \times 21}{21} = 85$.

25. Solution: 6.
$\overline{a2011b}$ is divisible by 12 . So it is divisible by 4. So the last two digits must be divisible by 4. Thus b can be 2 or 6.

The sum of the digits $\overline{a2011b}$ must be divisible by 3.

For $b = 2$, $a + 2 + 0 + 1 + 1 + 2 = a + 6$. So a can be 3, 6, and 9.
For $b = 6$, $a + 2 + 0 + 1 + 1 + 6 = a + 1 + 9$. So a can be 2, 5, and 8.
The answer is 6.

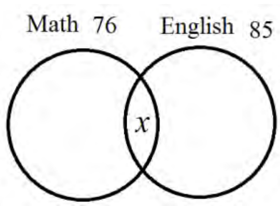
26. Solution: 9.
Let n be the number of sides on the convex n-gon, and m be the measure of the exterior angle.
We know that $(n - 2) \times 180 + m = 1350 \quad \Rightarrow \quad 180n + m = 1710$
We use the following way to solve it: $m = 1710 \mod 180$
We also know that $m < 180$.
So $m = 90$, and $n = 9$.

27. Solution: 8.
Case I: When these is no switch off:
ON ON ON ON

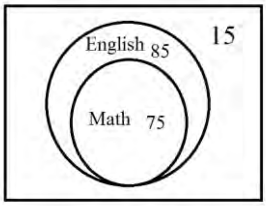
Case II: When these is one switch off:
ON ON ON OFF
ON ON OFF ON
ON OFF ON ON
OFF ON ON ON

Case III: When there are two switches off:
ON OFF ON OFF
OFF ON OFF ON
OFF ON ON OFF

28. Solution: 130.
$100 = 75 + 80 - x \quad \Rightarrow \quad x = 55$

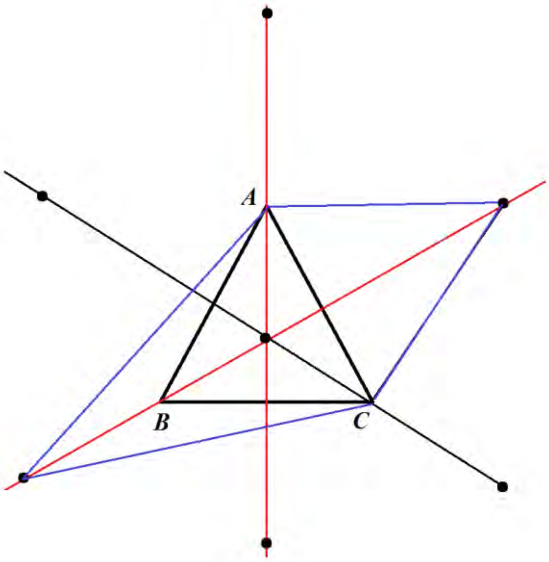


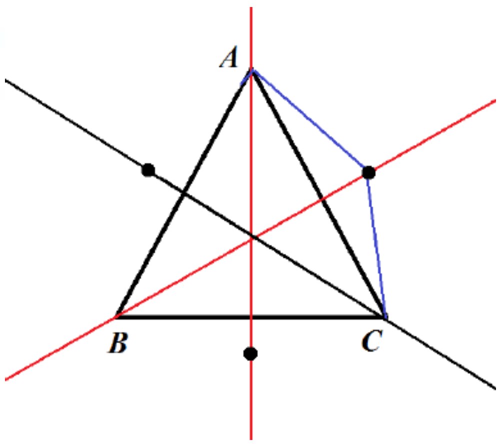
At most we can have 75 students got perfect scores on both subjects (assuming 15 students did not get a perfect score on any subject.



So the answer is 55 + 75 = 130.

29. Solution: 10





30. Solution: $\dfrac{\sqrt{5}+1}{2}$.

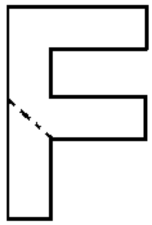
$a^2 = b(b+a) \Rightarrow (\dfrac{b}{a})^2 + \dfrac{b}{a} - 1 = 0 \Rightarrow \dfrac{b}{a} = \dfrac{-1+\sqrt{5}}{2} \Rightarrow a = 1, b = \dfrac{\sqrt{5}-1}{2}$.

$a + b = \dfrac{\sqrt{5}+1}{2}$.

1. Solution:

$$\left(\frac{37}{7}+\frac{37}{9}\right)\div\left(\frac{10}{9}+\frac{10}{7}\right)=\frac{37(\frac{1}{7}+\frac{1}{9})}{10(\frac{1}{9}+\frac{1}{7})}=\frac{37}{10}.$$

2. Solution:



3. Solution:

$(19 + 91) + (28 + 82) + (37 + 73) + (46 + 64) + 55 + x = 550 \Rightarrow 455 + x = 550 \Rightarrow x = 100$.

4. Solution: 81420.

The units digit of the subtrahend must be 0. We see that 93765 – 81420 = 12345.

5. Solution: 130.

We use the back calculation. Before Alex takes the marbles the 7th times, there are (3 – 1) × 2 = 4 marbles left. There were 130 marbles originally in the bag.

$\boxed{7}$ $\boxed{6}$ $\boxed{5}$ $\boxed{4}$ $\boxed{3}$ $\boxed{2}$ $\boxed{1}$

$3 \leftarrow 4 \leftarrow 6 \leftarrow 10 \leftarrow 18 \leftarrow 34 \leftarrow 66 \leftarrow 130$

6. Solution: 123.

It takes (4 – 1) × 30 = 90 seconds for Catherine to gets to 4th floor. It takes (15 – 4) × (30 ÷ 10) = 33 seconds.

The answer is 90 + 33 = 123 seconds.

7. Solution: 36.

Note the bottom faces are not painted. We view the solid from the top and we see 10 faces. We view the solid from right and we see 6 faces. We view the solid from front and we see 7 faces. The answer is $10 + (6 + 7) \times 10 = 36$.

8. Solution: 18.

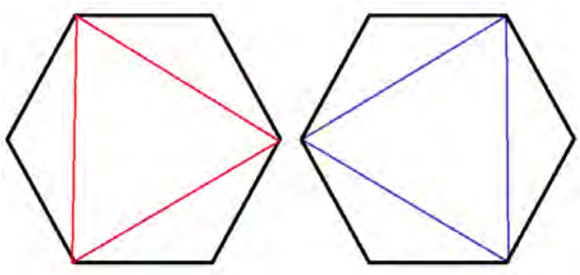
When we fold the paper twice, we get 4 pieces with 3 creases. When we cut the paper, each crease will generate a 2 by 4 rectangle, and other pieces will be square. So we get $48 \div 2 - 3 \times 2 = 18$ squares.

9. Solution: 1887.

The first digit is 1. The sum of the rest three digits is 23 with each digit greater than 5. $2013 - 100 = 1913$. It is easy to get the year 1887.

10. Solution: 18.

The number of triangles is $\binom{6}{3} = 20$. Two of them have no common sides with the hexagon. The answer is $20 - 2 = 18$.



11. Solution: 406.

$10 + 99 \times 4 = 406$.

12. Solution: 16.

We know that Karen's mother is older than 42, and in fact it is 43. Let Karen's age be x. Then we have $(4200 + x) - (43 - x) = 4289 \quad \Rightarrow \quad x = 16$.

13. Solution: 13.
When 1 is added to a counting number, the result is a multiple of 2. So we know the original number is odd. Also it has the remainder 1 when it is divided by 3. So we can have : 7, 13, 19….Then we check to see which one is a multiple of 5 when 1 is added to the triple of the number. 13 is the answer.

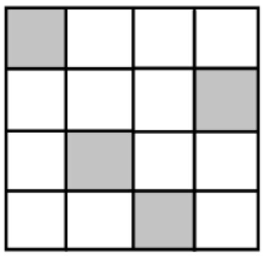
14. Solution: 24.
Let the number of boys be b and the number of girls be g.

$$b = 2g \qquad (1)$$

$$2(b - 12) = g \qquad (2)$$

Substituting (1) into (2): $g = 8$. So $b = 16$. The answer is $8 + 16 = 24$.

15. Solution: 2.



16. Solution: Bob.
We find two people with statements that contradict to each other: Alex and Catherine. We know that only one student is correct. So it must be either Alex or Catherine. Thus what Bob said is not true. So Bob can drive.

17. Solution: 550.

$$((3\times1+7\times1)+(3\times2+7\times2)+\cdots+(3\times10+7\times10)=10\times1+10\times2+\cdots+10\times10=550$$

18. Solution: 365 days.

$$(10\times10+11\times11+12\times12+13\times13+14\times14)+7\times10)\div2=365$$

19. Solution: 764.
We have (108 – 100) ÷ (100 – 99) = 8 numbers in addition to 108. So we have 9 numbers. The greatest value of the largest number is 9 × 100 – 108 – 1 – 2 – 3 – 4 – 5 – 6 – 7 – 8 = 764.

20. Solution: 9/4.
Since ΔABE and ΔADE have the same height, $\frac{S_{\Delta ADE}}{S_{\Delta ABE}} = \frac{DE}{EB} = \frac{3}{2}$.

Since ΔABE and ΔCDE are similar, $\frac{S_{\Delta CDE}}{S_{\Delta ABE}} = (\frac{DE}{EB})^2 = \frac{9}{4}$.

21. Solution: 51.
We divide these 20 numbers into two groups. The smallest sum is 1 + 2 + ··· + 10 = 55 and the greatest sum is 11 + 12 + ··· + 20 = 155.

55 × 155, 56 × 154, ···, 103 × 105, 105 × 105. Total we get 155- - 105 + 1 = 51 different products.

22. Solution: 48 km/hour.
At the same time, train A travels 120 + 30 = 250 and train B travels 280 – 10 = 150 m. So the ratio of the speeds of train B to train A is 150/250 = 3/5.
So the speed of train B is $80 \times \frac{3}{5} = 48$ km/hour.

23 Solution: 35 hours.
The shortest route will be T C D E A B T.

The time needed is 2 + 7 + 3 + 9 + 9 = 35 hours.

24. Solution: 15.
The ratio of the number of TV sold before and after the price is marked down is 1:2 and the ratio of the profits is 1:1.5 = 2:3. So the ratio of the profit of each TV is $\frac{1}{2} : \frac{3}{2} = \frac{4}{3}$.

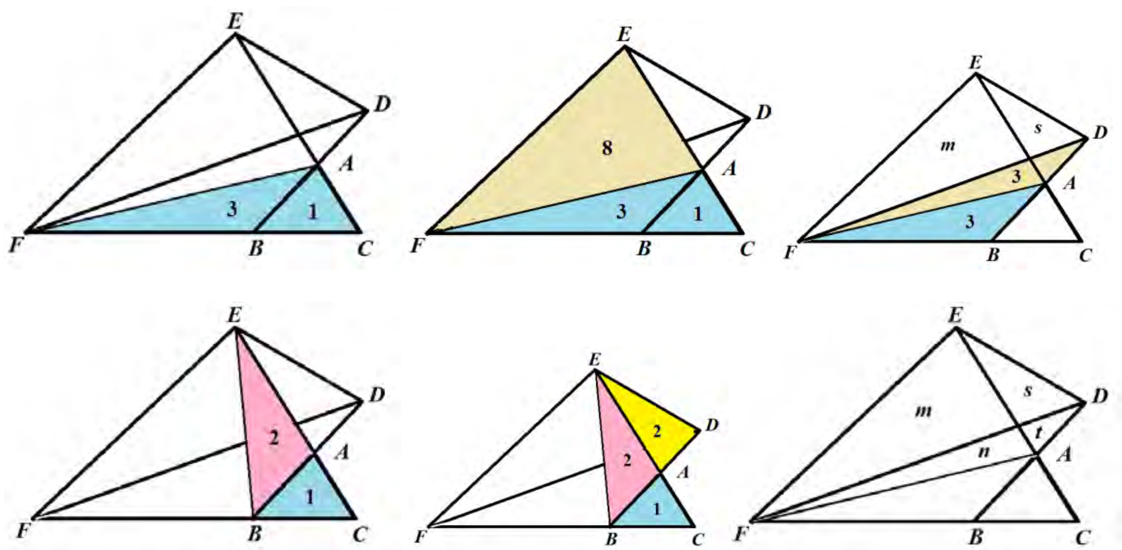
Thus the price of each TV after the price is marked down is $60 \times \frac{3}{4} = 45$. So 60 – 45 = 15.

25. Solution: 6.
We have 1000 digits from 2010 to 1761. So the 999th digit is 6.

26. Solution: 42.
If 50 positive integers are all even, the sum will be 2 + 4 + 6 + …+ 100 = 2550. Then we change some to be odd.

We change 100 to be 1. So the sum is 99 less. We change 98 to be 3. The sum is 95 less. We keep doing this until we change 90 to be 11. The sum is then 2550 – 99 – 95 – 91 – 87 – 83 – 79 = 2016. Now we change at least two more even numbers to get the sum to be 2010. So the answer is 42.

27. Solution: 7.



$m + n = 8$ (1)
$n + t = 3$ (2)
$s + t = 2$ (3)

$m + s = 7$

28. Solution:
Let a and b be two positive integers with $a < b$.

$a + 11 = b$.

Let the sum of the digits of a be $11x$. Then the sum of the digits of b will be $11x + 1 + 1 - 9y = 11x + 2 - 9y$ (y is the number of carries of the sum of a and 11).

Since the sum of the digits of b is a multiple of 11, the smallest possible value for x is 9 and for y is 10. That is, the number of carries is at least 10. The sum of the digits of a is at least 99. Therefore the smallest possible value of a is 189999999999.

29. Solution: 6.5 km/hour.
Let V_a and be Alex's speed and V_b be Bob's speed. Let d be the distance from village A to village B.

$$(V_A + V_B) \times 8 = d \qquad (1)$$

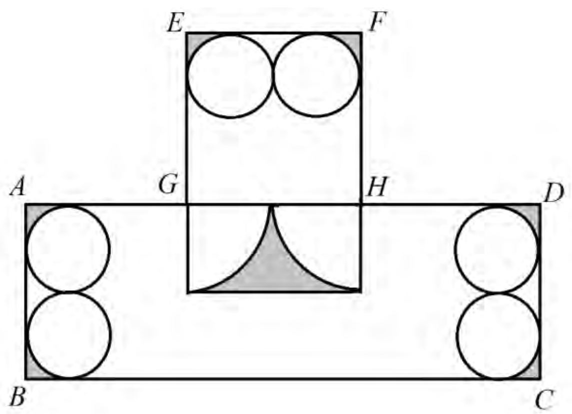
$$(V_A + 2 + V_B + 2) \times 6 = d \qquad (2)$$

$$(V_A + 2) \times 6 = \frac{d}{2} + 3 \qquad (3)$$

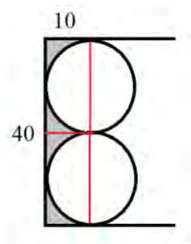
Solving the system of equations (1) and (2) we get $V_B = 12 - V_A,\ d = 96$.

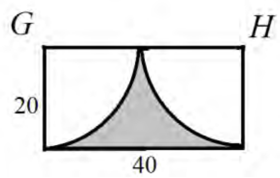
Substituting these values into (3): $V_A = \frac{39}{6} = 6.5$.

30. Solution:
Total area brushed by rolling the circle is the sum of the area of the 40 by 160 rectnagle and 40 by 40 sqaure minus the shaded areas as shown in the figure.



We see 6 congruent small shaded areas. Each has the area of $\frac{40\times 10-\pi\times 10^2}{4}$. The area of six of them will be $\frac{40\times 10-\pi\times 10^2}{4}\times 6=600-150\pi$.





One big shaded area is $20\times 40-2\times\frac{\pi\times 20^2}{4}=800-200\pi$.

The answer is $40\times 160+40\times 40-(600-150\pi)-(800-200\pi)=6600+350\pi=7699$.

Solutions to Practice Test 3

1. Solution: 42.
89 + 42 – 38 – 51 = 89 + 42 – 89 = 42.

2. Solution: $60.
They buy every three hamburger and pay $20 each. They pay $20 × 3 = $60 to buy 9 hamburger.

3. Solution: 24 days.
The hen lays one egg per day. So Bob consumes 3 eggs from 72 eggs each day. 72/3 = 24.

4. Solution: 800.
Since the sum is 1000, one of the numbers must be 888 (88 × 5 < 1000).
1000 – 888 = 112. So one of the number must be 88.
112 – 88 = 24 = 8 + 8 + 8.
The difference is 888 – 88 = 800.

5. Solution: 12351.
Φ is the number of 1's on the right.
Δ is *n* - 1 digit number 123……
Δ = 12345 and Φ = 6. The sum is 12345 + 6 = 12351.

6. Solution: 16.9

$$\frac{2010}{1000}+\frac{1219}{100}+\frac{27}{10}=2.01+12.19+2.7=16.9$$

7. Solution: 100.
(2400 × 5) ÷ (4 × 30) = 100.

8. Solution: 75.
The price after discounts is 3/4 of the original price. So the number of pens he can get with discount is 4/3 of the original.

The number of pens he could have bought with his money originally before discount is $25 \div (\frac{4}{3} - 1) = 75$.

9. Solution: $1200 + 300\pi$.

$(40^2 + \pi \times 20^2) - (20^2 + \pi \times 10^2) = 1200 + 300\pi$

10. Solution: 2010.

$4.02 = \frac{402}{100} = \frac{201}{50}$.

The 2-digit number must be 50. So $\frac{201}{50} \times 50 = 201$. The answer is 201 × 10 = 2010.

11. Solution: Saturday.

The first 28 days contain 4 weeks. So we have 4 Saturdays and 4 Sundays. We have two days left and one is Saturday and one is Sunday. So Saturday, April 29th is the same day as April 1st .

12. Solution: 280.

With two darts, Charles can get the following scores:

0, 1, 2, 3, 4, 8, 9, 11, 12, 13, 15, 16, 20, 23, 24, 26, 31, 35, and 46.

With three darts, he can get 5 = 1 + 4 + 0; 6 = 3 = 3 + 0; 7 = 3 + 3 + 1. Similarly we can get all the numbers up to 21 (12 + 8 + 1). We are not able to get 22.

13. Solution: 280.

Let x be the weight for the ball and y be the weight for the triangle block.

$x = 2y + 40$ (1)

$x + 80 = y + 200 \quad \Rightarrow \quad x = y + 120$ (2)

Substituting (1) into (2): $y = 80$. So $x = 200$.

$x + y = 200 + 80 = 280$.

14. Solution: 30.

$$\begin{array}{r} ABCD \\ +\ \ EFG \\ \hline 2\,0\,1\,0 \end{array}$$

So we see that $D + G = 10$; $C + F = 10$; $B + E = 9$; and $A + 1 = 2$.
So $A + B + C + D + E + F + G = 30$.

15. Solution: 8.
The difference between the number of boys and the number of girls is 40 – 0 = 40 originally. Each time after 3 boys are removed and 2 girls are added, the number of boys is 5 less. 40 ÷ = 8. So the answer is 8.

16. Solution: 18.
Assume that they finished x games already. The number of winning games is $x\times\frac{45}{100}$.
If his team win 6 out of the next 8 games, the number of games played is (x + 8).

$$(x+8)\times\frac{5}{100}=x\times\frac{45}{100}+6 \quad\Rightarrow\quad x=40.$$

The number of games Bob's team has already won is $x\times\frac{45}{100}$ = 18.

17. Solution: 201.

$$2010\Delta 2010=\frac{2010\times 2010}{2010+2010}=\frac{2010}{2}$$

$$\frac{2010}{2}\Delta 2010=\frac{\frac{2010}{2}\times 2010}{\frac{2010}{2}+2010}=\frac{2010}{3}$$

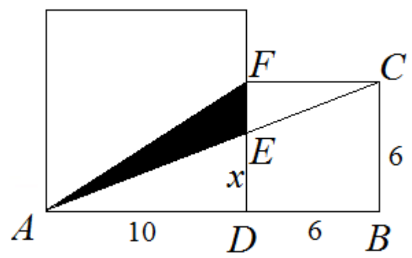
$$\frac{2010}{3}\Delta 2010=\frac{\frac{2010}{3}\times 2010}{\frac{2010}{3}+2010}=\frac{2010}{4}.$$

We see the pattern now and the answer is $\frac{2010}{10}\Delta 2010=\frac{2010}{10}=201$.

18. Solution: $\frac{45}{4}$.

Triangles *ABC and ADE* are similar. $\frac{6}{10+6}=\frac{x}{10} \quad\Rightarrow\quad x=\frac{60}{16}=\frac{15}{4}$.

The shaded area is $\frac{(6-x)\times 10}{2}=\frac{45}{4}$.



19. Solution: 56.
Let two numbers be a and b, respectively. We know that $a = 7m$ and $b = 7n$, where m and n are positive integers relatively prime.
We have $a + b = 70 \quad\Rightarrow\quad 7m + 7n = 70 \quad\Rightarrow\quad m + n = 10$.
Since we want the greatest possible difference of two numbers, we set them as far as possible by letting $m = 9$ and $n = 1$.
$7m - 7n = 63 - 7 = 56$.

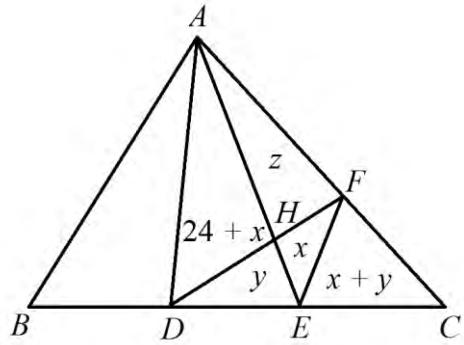
20. Solution: 3/5.

$$P=\frac{4!+\binom{2}{2}4!}{5!}=\frac{3}{5}.$$

21. Solution: 1.
We see that a and b are two roots of the equation: $x^2+x=1 \quad\Rightarrow\quad x^2+x-1=0$.
We also see that $a^2b+ab^2 = ab(a+b)$, where ab is the product of the roots and $a + b$ is the sum of the roots.
By Vieta's Theorem, for $x^2+x-1=0$, we have $a + b = -1$ and $ab = -1$. So $a^2b+ab^2 = ab(a+b)=1$.

22. Solution: 108 cm^2.
We label the areas as shown in the figure.
We know that the areas of ΔADE and ΔAEC are the same.
So we have: $24+x+y=z+x+x+y \quad\Rightarrow\quad x+z=24$ (1)
We know that the area of ΔADF is two times of the area of ΔCDF.
So we have: $24+x+z=2(x+y+x+y) \quad\Rightarrow\quad x+y=12$ (2)
Thus the area of ΔAEC is $z+x+x+y=24+12=36$.

We also know that the area of ΔAEC is one-third of the area of ΔABC.
So the answer is $36 \times 3 = 108 \text{ cm}^2$.



23. Solution: 12.
Let $n = 2^a 3^b$. The number of factors is $(a + 1)(b + 1)$.
$2n = 2 \times 2^a 3^b = 2^{a+1} 3^b$. The number of factors is $(a + 2)(b + 1)$.
$3n = 2^a 3^{b+1}$. The number of factors is $(a + 1)(b + 2)$.
We know that $(a + 2)(b + 1) - (a + 1)(b + 1) = 2 \quad \Rightarrow \quad b = 1$.
We know that $(a + 1)(b + 2) - (a + 1)(b + 1) = 2 \quad \Rightarrow \quad a = 2$.
The positive integer is $n = 2^a 3^b = 2^2 3^1 = 12$.

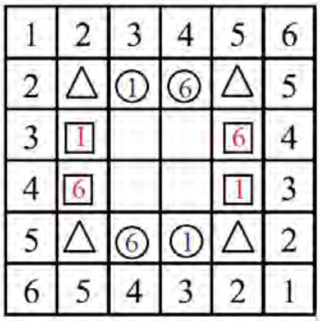
24. Solution: 16.
We see that the number in each small square must be 1 or 6. We have two ways to fill them.

1	2	3	4	5	6
2					5
3	$\square$			$\square$	4
4	$\square$			$\square$	3
5					2
6	5	4	3	2	1

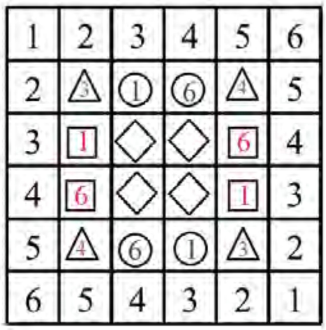
1	2	3	4	5	6
2					5
3	$\boxed{6}$			$\boxed{1}$	4
4	$\boxed{1}$			$\boxed{6}$	3
5					2
6	5	4	3	2	1

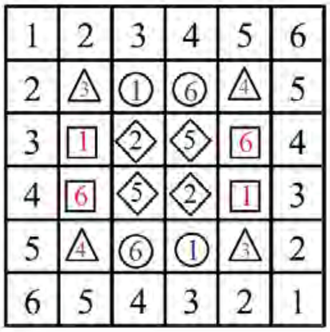
1	2	3	4	5	6
2					5
3	$\boxed{1}$			$\boxed{6}$	4
4	$\boxed{6}$			$\boxed{1}$	3
5					2
6	5	4	3	2	1

After we fill the four squares, we see that the number in each circle must be 1 or 6 as



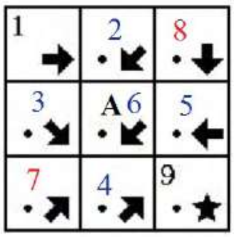
well. We have two ways to fill them.
Similarly we have two ways to fill four small triangle (3 or 4) and two ways to fill the four rhombuses (2, 5).



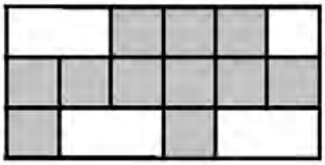


Total we have 2 ×2 ×2 ×2 = 16 ways.

25. Solution: 6.
We try several times and we see that 7 must be in the left bottom square and 8 must be in the right top square. A must be 6.

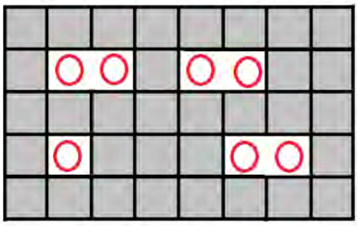
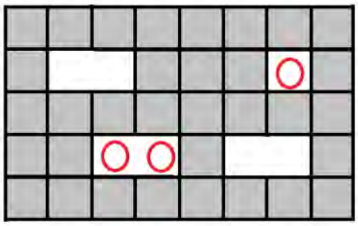


26. Solution: 30.
We see that all the shaded small squares along the perimeter of the rectangle. We get 5 ×2 + 6 × 2 = 22.
Now we remove all the squares already counted and we get the following figures:

It is easy to see that we have 8 more small squares overlapping. The answer is 22 + 8 = 30.

Method 2: We count the number of squares that are not shaded and we get 10 of them. The answer will be $5 \times 8 - 10 = 30$.

27. Solution: 2.
In order to get the least number of questions every one answered correctly, we need to let the number of questions they answered wrong to be different. So at most we get $2 \times 4 = 8$ problems answered wrong. Thus we get at least $10 - 8 = 2$ questions they all get correct.

28. Solution: 13.
These numbers are in order
1 9 17 14 11 8 16 13 10 7 15 12 9,..
Starting from 9, the pattern repeated every 11 numbers.
$2009 \div 11 = 182$ r 7. So 2010^{th} number is the same as the seventh number is the pattern, which is 13.

29. Solution: Friday.
From the given conditions, if we use X to indicate that the flower does not bloom, we have:

	M	T	W	TH	F	SA	SU
A		x		x			x
B				x		x	
C							x

From (3), we know that B must bloom on Tuesday and Sunday, and C must bloom on Tuesday and Thursday.

	M	T	W	TH	F	SA	SU
A		x		x			x
B		✓		x		x	✓
C		✓		✓			x

Since no flower can bloom in 3 consecutive days, C will not bloom on Wednesday. From (3) again, A must bloom on Wednesday.

	M	T	W	TH	F	SA	SU
A		x	✓	x			x
B		✓		x		x	✓
C		✓	x	✓			x

and B will bloom on Wednesday (if not we will get contradiction).

	M	T	W	TH	F	SA	SU
A		x	✓	x			x
B		✓	✓	x		x	✓
C		✓	x	✓			x

From (1) again, we know that B will not bloom on Monday:

	M	T	W	TH	F	SA	SU
A		x	✓	x			x
B	x	✓	✓	x		x	✓
C		✓	x	✓			x

Which leaves only Friday (and it works).

30. Solution: 11.

Let r be the radius of the cylinder. Then the height is $3r$. The area of the base is πr^2.

The lateral area is $3r \times 2\pi r = 6\pi r^2$.

So the total surface area of two cylinders after cutting is $\pi r^2 \times 4 + 6\pi r^2 = 10\pi r^2$.

The surface area of the smaller cylinder is $10\pi r^2 \times \frac{1}{1+3} = \frac{5}{2}\pi r^2$.

The lateral area of the smaller cylinder is $\frac{5}{2}\pi r^2 - 2\pi r^2 = \frac{1}{2}\pi r^2$.

The height of the smaller cylinder is $\frac{\frac{1}{2}\pi r^2}{2\pi r} = \frac{1}{4}r$.

The height of the larger cylinder is $3r - \frac{1}{4}r = \frac{11}{4}r$.

The height of the larger cylinder is $\frac{11r}{4} \div \frac{1}{4}r = 11$ times of the height of the smaller cylinder.

Since both cylinders have the same area of the base, the volume of the larger cylinder is 11 times of the volume of the smaller cylinder.

Solutions to Practice Test 4

1. Solution: 14.
Method 1:
$(11 + 12 + 13 + 14 + 15 + 16 + 17) \div 7 = [7 \times (11 + 17) \div 2] \div 7 = 14.$
Method 2:
$(11 + 12 + 13 + 14 + 15 + 16 + 17) \div 7 = [7 \times 14] \div 7 = 14.$

2. Solution: 25.
The rule is this: a∆b: starting with a, each time you add 1 and write b numbers.
$3\Delta5 = 3 + 4 + 5 + 6 + 7 = (3 + 7) (4 + 6) + 5 = 25.$

3. Solution: 8.
Let the number of \$5 bills be x. Then $5x + 10(14 - x) = 100 \quad \Rightarrow \quad x = 8.$

4. Solution: 2160.
$10 + 11 + \ldots + 99 = (11 + 99) \times 45 \div 2 = 2475.$
Among them, the following are divisible by 9: 27, 45, 63, 81, and 99. The sum of the five numbers is $(27 + 99) \times 5 \div 2 = 315.$
The answer is $2475 - 315 = 2160.$

5. Solution: Saturday.
We know that August has 31 days. So we start with 31st day and put all five Mondays into the calendar such that we exactly get four Tuesdays as shown.

M	T	W	Th	F	Sat	Sun
3	4					
10	11					
17	18					
24	25					
31						

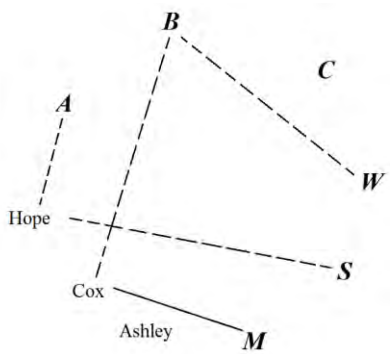
So we know that August 3rd is a Monday. Then we conclude that August 8 is a Saturday.

M	T	W	Th	F	Sat	Sun
3	4	5	6	7	8	
10	11					
17	18					
24	25					
31						

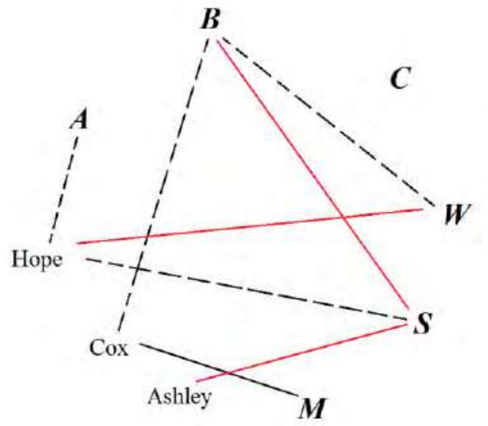
6. Solution: 6.
The sum of four digits is $2 + 5 + 7 + 9 = 23$. When 23 is divided by 3, the remainder is exactly 2. So when we select 3 numbers and leave one number, the one left must be divisible by 3. We see 9 must be the number left out.
We have $3! = 6$ such 3-digit numbers with the arrangement of the digits 2, 5, and 7.

7. Solution: 720.
In any hour, the hour hand turns $30°$, the minute hand turns $360°$, and the second hand turns $360 \times 60° = 21600°$. The ratio is $21600/30 = 720$.

8. Solution: Hope School and Writing Club.
We use dash line to indicate a no relationship and a solid line a yes relationship. The figure below shows the information we are given.



From figure, we know that the student from hope School must be in writing club. The student from Ashley School must be in Science Club. Bob must be in Science Club. Therefore Bob is from Ashley School. Since Alex is not from Hope School, so he is from Cox School. Thus Charlie is from Hope School and is in Writing Club.



9. Solution: 4.

In order for at least one kid gets 7 apples or more, the number of apples is at least (7 – 1) times of the number of kids plus 1. We see that $4 \times (7 - 1) + 1 = 25$. So n must at most be 4.

10. Solution: 4.

Let the number of students who did not take any exam be x.

Using Venn diagram, we have: $40 - x = 32 + 24 - 20 \quad \Rightarrow \quad x = 4$.

11. Solution: 46.

Method 1:

Figure 1 has $4 + 1 \times 2 = 6$ small circles.

Figure 2 has $4 + 2 \times 3 = 10$ small circles.

Figure 3 has $4 + 3 \times 4 = 16$ small circles.

…

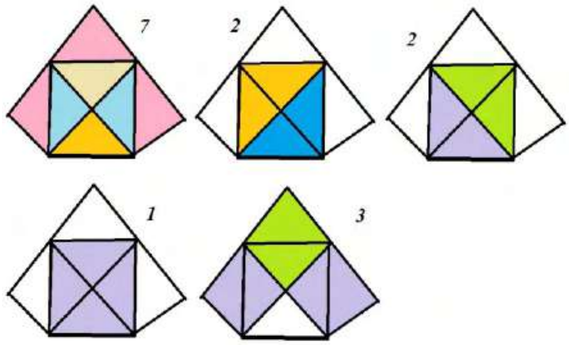
Figure 6 has $4 + 6 \times 7 = 46$ small circles.

Method 2:

Figure	1		2		3		4		5		6
# of circles	6		10		16		24		34		46
		4		6		8		10		12	
			2		2		2		2		

12. Solution: 2500 km.
The distance from Apex to Burnham is 2 cm and the distance from Apex to Cali is 3 cm. So the greatest distance from Apex to Cali is 2 + 3 = 5 cm. The actual distance is then5 × 50,000,000 = 250,000,000 cm = 2500 km.

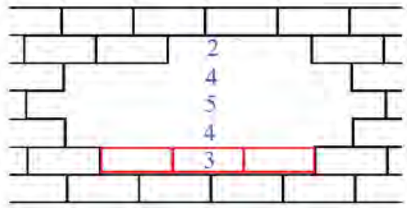
13. Solution: 15.
7 + 2 + 2 + 1 + 3 = 15.



14. Solution: 150.
The distance the big rabbit runs 5 steps is the same as the distance the small rabbit runs 7 steps. So the distance the big rabbit runs 5 × 3 = 15 steps is the same as the distance the small rabbit runs 7 × 3 = 21 steps.
We know that the time used by the small rabbit to run 4 steps is exactly the same as the time used by the big rabbit to run 3 steps. So when bog rabbit runs 15 steps, small rabbit runs 20 steps. So evey 15 steps the big rabbit runs, it gets 1 small rabbit stpe closer to small rabbit. Therefore it takes 15 × 10 = 150 steps for the big rabbit to catch the small rabbit.

15. Solution: 25.
They can use 15 empty cans to exchange for 15/3 = 5 can of soda. Now they have 2 empty cans and 5 cans of soda. When they finish 5 cans of soda, they have 7 empty cans. They can use 6 of them to exchange for two cans of soda. When they finish 2 cans of soda, they have 3 empty cans . They use these 3 empty cans tog et one more can of soda. Total 17 + 5 + 2 + 1 = 25.

16. Solution: 18.

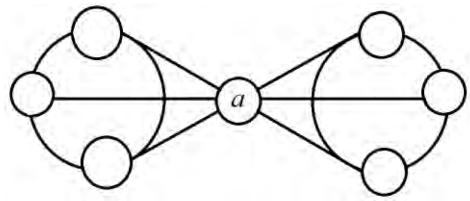


17. Solution: 12.

Let a be the number in the circle where three lines meet. Let k be the sum of 3 numbers in each line.

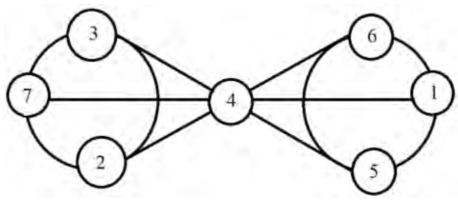
So we have $2(1 + 2 + 3 + 4 + 5 + 6 + 7) + a = 5n$

$\Rightarrow \quad 56 + a = 5n \quad \Rightarrow \quad 55 + a + 1 = 5n.$



So $a + 1$ must be divisible by 5. We see that $a = 4$ works. And we get $n = 12$.

One such arrangement is followed.



18. Solution: 543279.

The smallest number should be a 2-digit number. It could be 78 or 69. So we use 69.

The greatest number must be the one with as many digits as possible with distinct digits.

We can get 543210. So the answer is 69 + 543210 = 543279.

19. Solution: 150.

The boy moves $20 \times 5 = 100$ stairs in 5 minutes.

The girl moves $15 \times 6 = 90$ stairs in 6 minutes.

The girl uses one more minute than the boy but travels $100 - 90 = 10$ stairs less than the boy.

So the escalator moves 10 stairs in this one minute.

In 5 minutes, the boy and the escalator move $(20 + 10) \times 5 = 150$ stairs. That is, there are 150 stairs visible when the escalator is still.

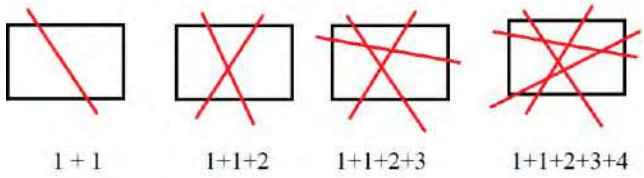
20. Solution: 18 days.

Two groups finished $2400 - 204 = 2196$. In each day they assembled $60 + 62 = 122$ sets. $2196/122 = 18$ days.

21. Solution: 22.

Method 1:

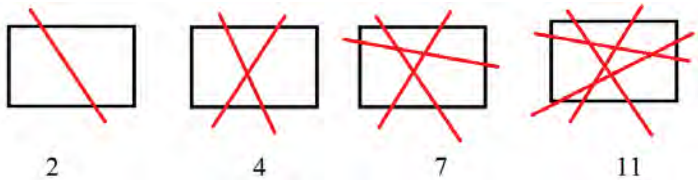
We draw several figures and count the number of regions.

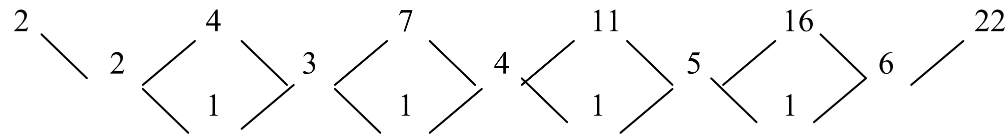


The pattern is $1+\frac{(1+n)n}{2}$. N is the number of lines. So $1+\frac{(1+6)6}{2}=22$.

Method 2:

We draw several figures and count the number of regions.





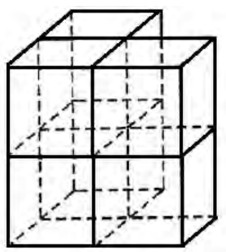
22. Solution: 61.

Let 3 numbers be a, $a + 2$, and $a + 4$. a is odd.

We see that $a + 4 - a = 4$.

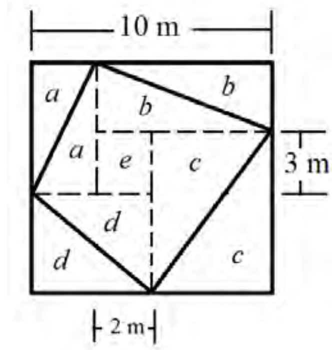
We are given that $(a + 2)(a + 4) - a(a + 2) = 252 \Rightarrow 4(a + 2) = 252 \Rightarrow (a + 2) = 63$. So the smallest number is $63 - 2 = 61$.

23. Solution: 6.



24. Solution: 53.

We label each region as shown in the figure.



$2a + 2b + 2c + 2d + e = 100$. We know that $e = 3 \times 2 = 6$.

So $a + b + c + d = 47$.

The shaded area $= a + b + c + d + e = 47 + 6 = 53$.

25. Solution: 3.

Let w be the weight of the water originally in the tank and t be the weight of the tank.

$2w + t = 10$ (1)

$5w + t = 19$ (2)

(2) - (1): $3w = 9 \quad \Rightarrow \quad w = 3$

26. Solution: 9 and 2.

$77 = 7 \times 11$. Since $m - n < m + n$, we have:

$m - n = 7$ and $m + n = 11$. So $m = 9$ and $n = 2$.

27. Solution: 126.

	Day 1	2	3	4	5	6
A	2	4	8	16	32	64
B	6	6	6	6	6	6
Sum	8	8+10=18	18+14=32	32+22=54	54+38=92	162

The number of centimeters Mouse A will have dug when they finally meet is

$2 + 4 + 8 + 16 + 32 + 64 = 126$.

28. Solution: 7.

If there are 3 students from class A getting into top 3, then there are 6 students from class B getting into top 3. Nobody in top 3 is from class C. Then there is no tie for the first place.

We also know that the total number of points is an odd number: $3(5 + 3 + 1) = 27$. Thus at most two students get into top 3 from class A. And these two students should both get first place so they can tie with the first place class. If only one student from class A gets

the first place, and the other student from class A gets the second place, they can only get 8 points which is less than the average point 27/3 = 9.
So class A gets 10 points and class B gets 10 points as well. Class C gets 7 points.

29. Solution: $28.
After the last exchange, Alex has 168/3 = $56.
After the first exchange, Alex has 56 ÷ 2 = $28.
This is the amount of money Alex has at the beginning than Bob.

30. Solution: 420.
Train A travels $20 - 14 = 6$ m more than the distance train B travels in a second.
When they start at the same time with their heads at the same starting points, train A passes train B completely in 40 seconds. So train A travels $6 \times 40 = 240$ m more than the distance travelled by train B. So train A is 240 meters long.

When train A and train B start at the same time with their tails at the same starting points, train A passes train B completely in 30 seconds. So train A travels $6 \times 30 = 180$ m more than the distance travelled by train B. So train B is 180 meters long.
The asnwer is 240 + 180 = 420 meters.

Solutions to Practice Test 5

1. Solution:
Method 1:
$(100 - 98) + (96 - 94) + (92 - 90) + \cdots + (4 - 2) = 2 + 2 + 2 + \cdots + 2 = 2 \times 25 = 50.$
Method 2:
$(100 + 96 + 92 + \cdots + 2) - (98 + 94 + 90 + \cdots + 2 = 25 \times (100 + 4) \div 2 = 1300 - 1250 = 50.$

2. Solution: 17.
Method 1:
2, 3, 4, 5,.. is an arithmetic sequence with common difference 1.
5, 7, 9, 11.. is an arithmetic sequence with common difference 2.
8, 11, 14, x,.. should be an arithmetic sequence with common difference 3.
So $x = 17$.

Method 2:
$8 = 2 + 5 + 1$
$11 = 3 + 7 + 1$
$14 = 4 + 9 + 1$
So $x = 5 + 11 + 1 = 17$.

3. Solution: \$3.
Method 1:
Two pounds of apples cost the same as 2.5 pounds of pears.
Twenty pounds of apples cost the same as 25 pounds of pears.
So 55 pounds of pears cost 132. One pound of pear costs \$2.4.
One pound of apples costs $(132 - 2.4 \times 30) \div 20 = \$3$.

4. Solution: Charlie.
We find two people with statements that contradict to each other: Alex and Charlie. Since we know that only one person speaks the truth, that person must be eight Alex or Charlie. So Bob did not tell the truth. Thus we know that Bob knows French. Charlie tells the truth.

5. Solution: 1233.

We first calculate the sum of these 10 numbers: $789 \times 10 = 7890$.

We know that the average of eight of those ten is 678. So the sum of these 8 numbers is $678 \times 8 = 5624$.

The average of the other two numbers is $(7890 - 5424) \div 2 = 1233$.

6. Solution: 5544.

The minuend is at most 9999 and the subtrahend is at least 1000.

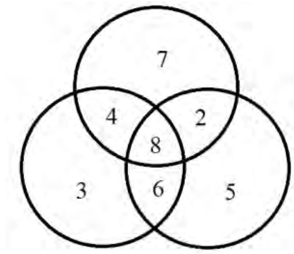
When the minuend is 9999, the greatest subtrahend can be $9999 - 3456 = 6543$.

When the subtrahend is 1000, the smallest minuend can be $1000 + 3456 = 4456$.

We see that $(9999 - 1) - (6543 - 1) = 3456$. Keep doing this we are able to get all the pairs. Therefore the number of pairs is $6543 - 1000 + 1 = 5544$.

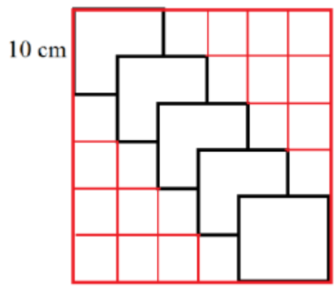
7. Solution: 8.

We see that each circle contains two even numbers already. In order to get an odd sum 21, each circle needs one even number and one odd number. So 8 must go to the middle.

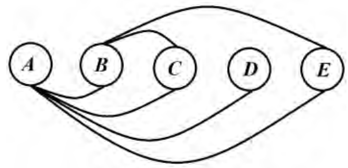


8. Solution: 120 cm.

We create a square as shown in the figure. The perimeter of the square is exactly yhe same as the perimeter we want to find. So the answer is $30 \times 4 = 120$.



9. Solution: 2.
We draw the chart and see that team D played 2 games.



10. Solution:
It is clear that the first place team is not A. Then we compare teams B and C.
From the figure, we see that team B's points on music is the same as team C's points.
Team B's points on art is the also same as team C's points on sport. However, Team B's points on sport is more than team c's points on art. So team B is the first.

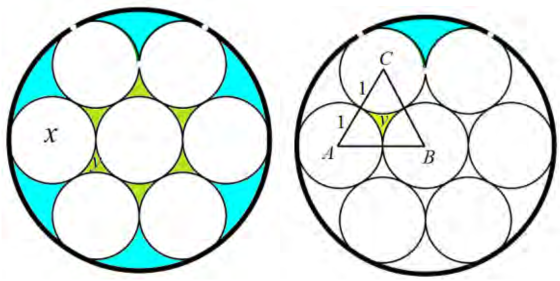
11. Solution: 5 meters per minute.
In 8 minutes, they walked $720 - 200 = 520$ m.
Since Alex's speed is 30 meters per minute, he walked $8 \times 30 = 240$ m
So Bob walked 520 - 240 = 280 m. Thus Bob' speed is $280 \div 8 = 35$ meters per minute.
The speed difference is 35 - 30 = 5 meters per minute.

12. Solution: 3400.
The number of days required to make 3000 toys is 15, 20, and 25 for Alex, Bob, and Charlie, respectively. So the number of toys Alex, Bob, and Charlie make is 200, 150, and 120, respectively.
The answer is $(200 + 150 + 120) \times 5 + (200 + 150) \times 3 = 3400$.

13. Solution: 75 m^2.
We know that there are 6 students in each row. So we get 96/6 = 16 students in a column.
We know that the distance is 1 meter between every two students. So width of the rectangle is $(6 - 1) \times 1 = 5$ m and the length is $(16 - 1) \times 1 = 15$ m. The area of the rectangle is $15 \times 5 = 75$ m^2.

14. Solution: $\frac{5\pi}{6} - \sqrt{3}$.
As shown in the figure, the shaded area = (the area of the large circle $- 7x - 6y$)/6.

$$= \frac{\pi \times 3^2 - 7 \times \pi \times (1)^2 - 6 \times (\frac{\sqrt{3}}{4} \times 2^2 - \frac{\pi \times (1)^2}{2})}{6} = \frac{5\pi}{6} - \sqrt{3}$$



15. Solution: 7.
If you ask for 1 marble, Alex only needs one bag with one marble in it.
If you ask for 2 marbles, Alex can prepare one more bag either with one marble in it or two marbles in it. $1 + 2 = 3$.

Alex needs a bag with 4 marbles in it to deal with the request for 5 or more marbles.
$4 + 1 = 5$, $4 + 2 = 6$. $4 + 2 + 1 = 7$.

Alex needs a bag with 8 marbles in it to deal with the request for 8 or more marbles.
We see that $127 = 1 + 2 + 4 + 8 + 16 + 32 + 64$.
So 7 is the smallest value for n.

16. Solution: 11.
Let x be the number of students who do not participate in any club.
$48 - x = 23 + 26 - 12 \quad \Rightarrow \quad x = 11.$

17. Solution: 3.
In 5 years, the sum of the ages increases by $5 \times 4 = 20$.
However, $83 - 65 = 18 < 20$.
So we know that Bob was not born 5 years ago.
So Bob is $5 - (20 - 18) = 3$ now.

18. Solution:

Step 1:	9	
	8	
Step 2:	9	3
	8	6
Step 3:	9	3

8 6 2

Step 4: $93 \times 862 = 80166$.

19. Solution: 100.

We divide the numbers by parity into two groups:

1	3	5	7	9
2	4	6	8	10

If we select one number from the first row and 3 numbers from the second row, we will get a sum of odd number.

If we select three numbers from the first row and one number from the second row, we will also get a sum of odd number.

So the answer is $\binom{5}{1} \times \binom{5}{3} + \binom{5}{3}\binom{5}{1} = 100$.

20. Solution: 210.

We see the pattern of the numbers in the first row:

1^{st} number: $1 = 1$

2^{nd} number: $3 = 1 + 2$

3^{rd} number: $6 = 1 + 2 + 3$

4^{th} number: $10 = 1 + 2 + 3 + 4$.

So the the 20^{th} number is $1 + 2 + 3 + \ldots + 20 = (1 + 20) \times 20 \div 2 = 210$.

21. Solution:

$\binom{4}{2} = 6$. We should have 6 numbers. So we know that there are two numbers the same.

Since $B < C$, we can get $A + B < A + C$.

$A + B < A + C < B + C < D + C$.

Since we know that two numbers among them are the same, the only way is $B + C = A + D$.

So the 6 numbers must be 21, 23, 24, 24, 25, 27.

Add A, B, C, and D in pairs we get: $3(A + B + C + D) = (21 + 23 + 24 + 24 + 25 + 27)$

$\Rightarrow \quad A + B + C + D = (21 + 23 + 24 + 24 + 25 + 27) \div 3$.

The average of A, B, C, and D is $(21 + 23 + 24 + 24 + 25 + 27) \div 3 \div 4 = 12$.

22. Solution: 10830.
We take away all the unit cubes that are painted. If no side is attached to a wall, the dimensions of the solid left will be 40 − 2, 20 − 2, and 16 − 1.
Now we know that one side is attached to a wall.
Case 1: if the side 40 × 16 is attached to the wall, the number of unit cubes no pained faces will be (40 − 2) × (20 − 1)× (16 − 1) = 10830.
Case 1: if the side 16 × 20 is attached to the wall, the number of unit cubes no pained faces will be (40 − 1) × (20 − 2)× (16 − 1) = 10530.
The answer is 10830.

23. Solution: 36.
The number of games played within each region is $\binom{4}{2} = 6$.
We have 4 regions. So the number of games is 6 × 4 = 24.
We have $2\binom{4}{2} = 12$ games in the final. So the answer is 24 + 12 = 36.

24. Solution: 52.
Since we want to put as many flowers as possible, we put one at each end. So we have (50 ÷ 2 + 1) × 2 = 52 flowers.

25. Solution: 340.
Note that
12341 ÷ 7 = 1763
1234112341 ÷ 7 = 176301763
123411234112341 ÷ 7 = 17630176301763
……

$$\underbrace{1234112341.....12341}_{\text{repeated 20 times}} \div 7 = \underbrace{1763017630\cdots1763}_{\text{twenty 1763's and nineteen 0's}}$$

So the answer is 20 × (1 + 7 + 6 + 3) = 340.

26. Solution: 1225 square units.
The side length of the white square brick is (45 + 25) ÷ 2 = 35.
So the area is 35^2 = 1225 square units.

27. Solution: Box 3.
We find two statements that contradict to each other: box 1 and box 4.
Since we know that only one note is correct, the note on box 3 will be false. So Candy is in box 3.

28. Solution: 14.
When a lamps' label is a square number, the number has odd number of factors. When the switch of the lamp is changed an odd number of times, the lamp will be "on". For example, the lamp labeled "16" will be switched by students numbered 1, 2, 4, 8, and 16. There are 14 square numbers from 1 to 200, with the smallest square number 1 and the greatest square number 14 = 196. So 14 lamps will show "on" position after every kid finishes.

29. Solution: 854.

$$\overline{abc}+\overline{acb}+\overline{bca}+\overline{bac}+\overline{cab}+\overline{cba}$$
$$=(a+b+c)\times 100\times 2+(a+b+c)\times 10\times 2+(a+b+c)\times 1\times 2$$
$$=(a+b+c)\times 222=3774.$$
$$a+b+c=\frac{3774}{222}=17.$$

The greatest one is 854.

30. Solution: tie.
Tommy needs 1000 ÷ 8 = 125 minutes to travel 100 meters.

The distance travelled by Robby in 5 minutes is 40 × 5 = 200 meters. Since Robby needs to rest for 25 minutes, so it will travel 200 meters in 30 minutes.
In the first four 30-minute periods, Robby travelled 200 × 4 = 800 meters. When Robby runs another 5 minutes, it will finish the race.
So Robby needs 125 minutes to reach the end.
The result of the race is a tie.

Solutions to Practice Test 6

1. Solution: 7/3.

$$\frac{7}{12}\div[\frac{1}{4}\div(\frac{1}{3}-\frac{1}{4})]\times 12 = \frac{7}{12}\times 12\div[\frac{1}{4}\div(\frac{1}{3}-\frac{1}{4})] = 7\div(\frac{1}{4}\div\frac{1}{12}) = 7\div(\frac{1}{4}\times 12) = \frac{7}{3}$$

2. Solution: 40%.

Let the original fraction be $\frac{b}{a}$. The new fraction is $\frac{b(1-25\%)}{a(1+25\%)} = \frac{3b}{5a}$.

$$\frac{\frac{b}{a}-\frac{3b}{5a}}{\frac{b}{a}}\times 100\% = \frac{1-\frac{3}{5}}{1}\times 100\% = \frac{2}{5}\times 100\% = 40\%.$$

3. Solution:

$$\frac{2}{7} = 0.\overline{285714}.$$

2011 = 335 × 6 + 1.

So the 2011st digit after the decimal point is the same as the first digit after the decimal point. In this case, it is 2.

4. Solution: 8.

If a number is divisible by 18, it must be divisible by both 2 and 9.

Since the given number is even, so it is divisible by 2.

The sum of the digits is $(2 + 1 + 1) \times n + 2 = 4n + 2 = 2(2n + 1)$.

The smallest value of n is 4.

5. Solution: $p = 2$ and $q = 5$.

We see that 35, 13, and 135 are all odd. So one of $35p$ and $13q$ must be even. We are told that both p and q are prime numbers, so we have two cases:

(1). $35p$ is even.

We have $p = 2$ and $35 \times 2 + 13q = 135$ $\Rightarrow$ $q = 5$.

(2). $13q$ is even.

We have $q = 2$ and $35p + 13\times 2 = 135$ $\Rightarrow$ $p = \frac{109}{35}$ (not a prime).

So the answer is $p = 2$ and $q = 5$.

6. Solution: 16.

$5*3=5\times 2-3\times 3+5\times 3=10-9+15=16$

7. Solution: $\frac{41}{45}$.

Method 1:

$(86-9\times 2)\div(9+8)=4$, $\frac{8\times 4+9}{9\times 4+9}=\frac{41}{45}$.

Method 2:

Let the denominator be d and the numerator be n.

$d + n = 86 \Rightarrow n = 86 - d$ (1)

$\frac{n-9}{d-9}=\frac{8}{9} \Rightarrow \frac{n-1}{d}=\frac{8}{9} \Rightarrow 9n-9=8d$ (2)

Substituting (1) into (2): $d= 45$ and $n = 41$.

The original fraction is $\frac{41}{45}$.

8. Solution: 47.

Let the number be $\overline{abc}$.

The number must be divisible by both 3 and 5. So we have two cases: c is 0 or 5.

Case I: $c = 0$.

The sum of the first two digits must be divisible by 3.

So $a+b=3$ = 1+ 2. We get 2 numbers (120, 210).

$a+b=6$ = 5 + 1 = 4 + 2. We get 4 numbers.

$a+b=9$ = 8 + 1 = 7 + 2 = 6 + 3 = 5 + 4. We get 8 numbers.

$a+b=12$ = 9 + 3 = 8 + 4 = 7 + 5. We get 6 numbers.

$a+b=15$ = 9 + 6 = 8 + 7. We get 4 numbers.

Case II: $c = 5$.

$a+b+5$ must be divisible by 3.

So $a+b=1$ = 1 + 0. We get one number (105).

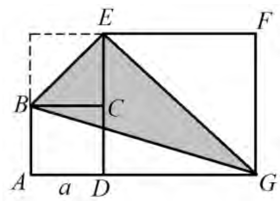
$a+b=4$ = 4 + 0 = 3 + 1. We get 3 numbers (405, 315, 135).
$a+b=7$ = 7 + 0 = 6 + 1 = 5 + 2 = 4 + 3. We get 7 numbers.
$a+b=10$ = 9 + 1 = 8 + 2 = 7 + 3 = 6 + 4. We get 8 numbers.
$a+b=13$ = 9 + 4 = 8 + 5 = 7 + 6. We get 6 numbers.
$a+b=16$ = 9 + 7. We get 1 number.
Total we get 2 + 4 + 8 + 6 + 4 + 1 + 7 + 8 + 6 + 1 = 47.

Note: This is 2010 China Hope Cup Math Contest training problem (6^{th} grade). The original answer was wrong. The question writer can only get 27 such numbers.

9. Solution: 12.
200 ÷ 13 = 15 r 5.
The quotient can be 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
When both the quotient and the remainder are 0, the dividend is 0 × 13 + 0 = 0.
So we can get 13 – 1 = 12 good numbers between 1 and 200.

10. Solution: 3.
We know that only (1), (2), or (3) can form a cube. The face labeled *A*, *B*, and *C* have a point of intersection. When we fold the net, A will face C in (1) and (2). So (3) is the answer.

11. Solution: 3.
Let the side length of the square *ABCD* be *a*.

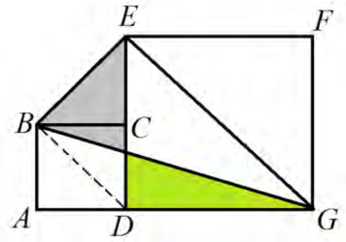


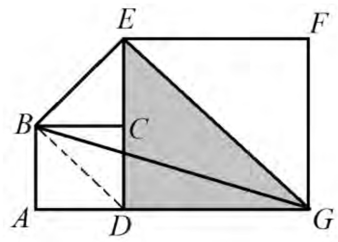
$$S_{\Delta BEG}=(a+a)(a+2a)-\frac{1}{2}\times 2a\times 2a-\frac{1}{2}\times a\times 3a-\frac{1}{2}\times a\times a$$

$$=2a\times 3a-2a^2-\frac{3}{2}a^2-\frac{1}{2}a^2=2a^2.$$

So $2a^2=6 \quad \Rightarrow \quad a^2=3$.

Method 2:
We connect *BD*. Since *BD*//*EG*, the area of triangle *BCE* is the same as the area of triangle *DCG*. So we know the area of the isosceles right triangle *EDG* is 6. So *ED* = $\sqrt{12}=2\sqrt{3}$. We are given that *ED* = 2*AB*. So $AB=\sqrt{3}$ and $AB^2=3$.





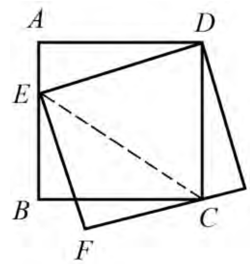
12. Solution:

The length of the rectangle is $48 \div 2 \times \frac{5}{5+3} = 24 \times \frac{5}{8} = 15$ cm.

The width of the rectangle is $48 \div 2 \times \frac{3}{5+3} = 24 \times \frac{3}{8} = 9$ cm.

The area of the rectangle is $15 \times 9 = 135\ \text{cm}^2$.

13. Solution: $\frac{144}{13}$



Connect CE. $S_{\Delta ECD} = \frac{1}{2} S_{ABCD}$

$S_{\Delta ECD} = \frac{1}{2} S_{DEFG}$

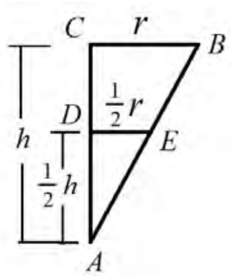
$S_{ABCD} = S_{DEFG} \quad \Rightarrow \quad 12 \times 12 = 13 \times DG \quad \Rightarrow \quad DG = \frac{144}{13}$.

14. Solution: 5:1.

The figure left is a sector with the central angle of 360° – 300° = 300°.

The ratio of the areas of the shaded portion to the sector that is cut off is 300°/60° = 5:1.

15. Solution: 21 liters.



The volume of the water is $\frac{1}{3}\pi \times (\frac{1}{2}r)^2 \times \frac{1}{2}h = \frac{1}{8} \times (\frac{1}{3}\pi r^2 h)$.

The number of liters of water needed to fill the cup to full is

$\frac{7}{8} \times (\frac{1}{3}\pi r^2 h) = 7 \times \frac{1}{8} \times (\frac{1}{3}\pi r^2 h) = 7 \times 3 = 21$.

16. Solution: 6 cm^3.
Let the length, width, and height be a, b, and c.
$a + b + c = 24 \div 4 = 6 \quad \Rightarrow \quad a + b + c = 3 + 2 + 1$.
The volume is $a \times b \times c = 3 \times 2 \times 1 = 6$ cm^3.

17. Solution: is 6:01 pm.
$2011 \div 60 = 33$ r 31
So the time taken for the second hand to turn 2011 revolutions is 33 hours and 31 minutes.

33 – 24 = 9.
8 : 30 plus 9 hours and 31 minutes is 6:01 pm.

18. Solution: $\frac{10}{3}$.
Let the rate for Alex be A and for Bob be B.

$$\frac{1}{A} \times 5 = 1 \qquad \Rightarrow \qquad \frac{1}{A} = \frac{1}{5} \qquad (1)$$

$$(\frac{1}{A} + \frac{1}{B}) \times 2 = 1 \qquad (2)$$

Substituting (1) into (2): $(\frac{1}{5} + \frac{1}{B}) \times 2 = 1 \Rightarrow \frac{1}{B} = \frac{1}{2} - \frac{1}{5} \Rightarrow \frac{1}{B} = \frac{3}{10} \Rightarrow B = \frac{10}{3}$.

19. Solution: 686 cm^3.
Let a be the length of the base square.
$a^2 = 98/2 = 49 \quad \Rightarrow \quad a = 7$
$2 \times 7 = 14$ will be the length of the prism.
So the volume is $7 \times 7 \times 14 = 686$ cm^3.

20. Solution: = $20.
The original price of shirt A is 0.16 ÷ 16% = $1.
The original price of shirt B is 3.36 ÷ 16% = $21.
The difference is 21 – 1 = $20.

21. Solution: 36.
Let the number of candy be $3x$.
$3x - 8 \times 3 = x \quad \Rightarrow \quad x = 12.$

Originally there are $12 \times 3 = 36$ pieces of canny.

22. Solution: 210 liters.
Let x be the number of liters in the beginning.
$[(1-10\%)x-59]\times(1-20\%)=\frac{1}{2}x-1 \quad \Rightarrow \quad (0.9x-59)\times 0.8=\frac{1}{2}x-1\Rightarrow$

$x=210$ liter.

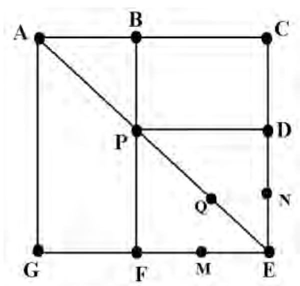
23. Solution:
Let a be the number of adult tickets and c be the number of children tickets.
$a + c = 99$ (1)
$3a + 2c = 280$ (2)
$3 \times (1) - (2)$: $c = \$17$.

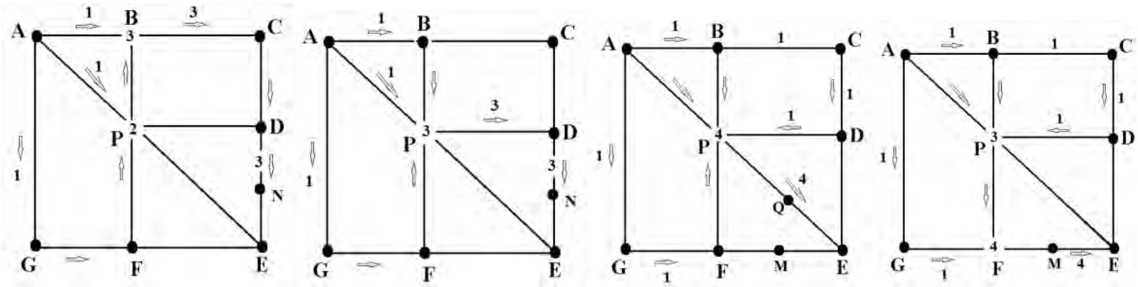
24. Solution: 12.
The number of girls less than 12 years old is $36\times(1-\frac{7}{12})(1-20\%)=36\times\frac{5}{12}\times 0.80=12$.

25. Solution: 6.
$2011 \div 4 = 502$ r 3. So the number of marbles in the last box is the same as the number of box in the third box, which is 6.

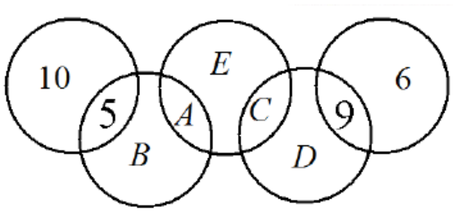
26. Solution: 14.
Every path from A to E must go through M, Q, or N as shown in the figure.

We count and get 14 paths.

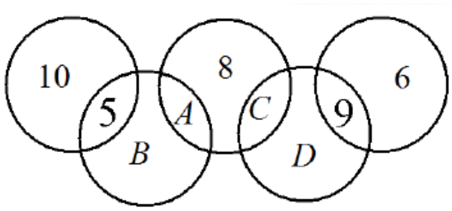




27. Solution: 2.
We see that the left most circle needs a 10 and the right most circle needs a 6.
The numbers left are 2, 3, 4, 7, and 8.



So $C + D = 6 = 2 + 4$.
So $A + B = 10 = 3 + 7$.
We conclude that $E = 8$.



We also see that $A + C + 8 = 15$. So $A + C = 7$. Thus $A = 3$ and $C = 4$. $D = 2$.

28. Solution: 62π m^2.
$4 \times \pi \div 2 \times 30 + \pi \times (4 \div 2) = 62\pi$.

29. Solution: 8.5 minutes.
The time needed for Alex to run from the front to the end is $\frac{800}{(3+1)\times 80} = 2.5$ minutes.

The time needed for Alex to run the end to the front is $\frac{800}{(3-1)\times 80}=5$ minutes.

The total time is 2.5 + 5 + 1 = 8.5 minutes.

30. Solution: *A, D, F*.
If a lamp is switched an even number of times, it does not change its on or off state.
If a lamp is switched an odd number of times, it changes from on to off or from off to on depending on the original on-off position.
$2011 = 7 \times 287 + 2$.

All lamps are switched 287 times except *A* and *B*, which is switched 288 times.
A and *B* no change, *CDEFG* change.

Before:
A (on) C (on) E (on) G (on) B (off) D (off) F (off)

After
A (on) C (off) E (off) G (off) B (off) D (on) F (on)

So lamps *A, D, F* are on.

Solutions to Practice Test 7

1. Solution: 1.

$$\frac{1}{10}+\frac{8}{10}+\frac{1}{10}=1$$

2. Solution: 123456.

$a+\overline{aa}+\overline{aaa}+...+\overline{aaaaaa}$ =

$$\frac{a}{9}\times 9+\frac{a}{9}\times 99+\cdots+\frac{a}{9}\times 999999=\frac{a}{9}(10-1)+\frac{a}{9}\times(100-1)+\cdots+\frac{a}{9}\times(1000000-1)$$

$$=\frac{a}{9}(10+100+\cdots+1000000-6)=\frac{a}{9}(1111110-6)=\frac{a}{9}\times 1111104=123456a.$$

3. Solution: 61.

The remainder is always 1 when M is divided by 3, 4, 5, and 6. The LCM of 3, 4, 5, 6 is 60. So the smallest value for M is 60 + 1 = 61.

4. Solution: 20.

Since $A \div 2011 = 2010\ r\ B$, $A = 2010 \times 2011 + B$. The greatest value of B is 2010. When the remainder is greatest, $A = 2010 \times 2011 + B = 2010 \times 2011 + 2010 = 2010 \times 2012 =$ 4044120. So the last two digits are 20.

5. Solution: 44.

The sum is divisible by 46 if the sum is divisible by both 2 and 23. Since even = even + even or odd + odd, two numbers selected have the same parity.

$2011 \div 23 = 87\ r\ 10$. There are 87 multiples of 23 from 1 to 2011. 87 = 44 + 43. So we have 43 multiples of 23 in the form of $x \times 23$, where x is an even positive integer. And we have 44 multiples of 23 in the form of $y \times 23$, where y is an odd positive integer.

So the greatest possible value for n is 44.

6. Solution: 316.

Starting from the first row, every two rows are counted as a group with 13 numbers.

$2011 \div 13 = 154\ r\ 9$.

So 2011 is in the 155th group and the 9th number. So m = 154 × 2 + 1 + 1= 310th row. n = 6 column the same column as the number 9 in).
The answer is 310 + 6 = 316.

7. Solution: 54.

$A \div B \div C = 6 \quad \Rightarrow \quad A \div B = 6C$

$A \div B - C = 15 \quad \Rightarrow \quad A \div B = 15 + C$

So we get $6C = 15 + C \quad \Rightarrow \quad C = 3$.

Thus $A \div B = 15 + 3 = 18 \quad \Rightarrow \quad A = 18B$

We are given that $A - B = 17 \quad \Rightarrow \quad 18B - B = 17 \quad \Rightarrow \quad B = 1$ and $A = 18$.

$A \times B \times C = 18 \times 1 \times 3 = 54$.

8. Solution: 17/28.

The number of fractions with the same denominator is 1, 2, 3, 4…

We also see that 1 + 2 + 3 + 4+ …+ 14 =(14 + 1) × 14 ÷ 2 = 105,

that 1 + 2 + 3 + 4+ …+ 13 =(13 + 1) × 13 ÷ 2 = 91.

So the 100th fraction has the denominator 14 × 2 = 38.

Since 105 – 100 = 5, 14 – 5 = 9.

So the numerator of the 100th fraction is 2 × 9 – 1 = 17.

So the 100th fraction is 17/28.

Note: This is a 2010 China Hope Cup Math Contest training problem (6th grade). The original answer was wrong. The question writer's answer was 9/28.

9. Solution: 15.

$$\binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 4 + 6 + 4 + 1 = 15.$$

10. Solution: 21 and 35.

Method 1:

735= 5 × 147 = 5 × 3 × 49 = 5 × 3 × 7 × 7

672= 2 × 2 × 2 × 2 × 2 × 3 × 7

So we can see that two numbers are 21 and 35.

Method 2:
Let two numbers be x and y.

$xy = 735$ (1)

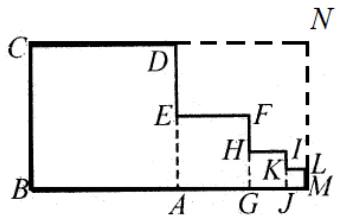
$x(y - 3) = 672$ (2)

(2) ÷ (1): $\frac{y-3}{y} = \frac{672}{735} = \frac{32}{35} \Rightarrow 1 - \frac{3}{y} = \frac{32}{35} \Rightarrow \frac{3}{y} = 1 - \frac{32}{35} = \frac{3}{35} \Rightarrow y = 35.$

Then $x = 21$.

11. Solution: 46 cm.
Method 1:
$AB = 8$. $EF = 4$. $HI = 2$. $IL = 1$.
Then perimeter is $(8 + 4 + 2 + 1) \times 4 - (4 + 2 + 1) \times 2 = 64 - 14 = 46$ cm.

Method 2:
The perimeter of $BCDEFHIKLM$ is the same as the perimeter of $BCNM$.
$2\times (BM + BC) = 2 \times (8 + 4 + 2 + 1 + 8) = 2 \times 23 = 46$ cm.

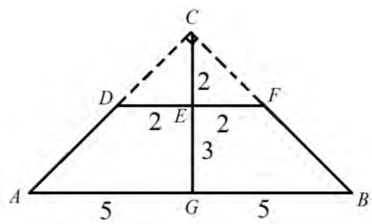


12. Solution: 5π cm^2.

Let r be the radius of the circle.

The shaded area is $\frac{1}{2} \times 2r \times r = 5 \Rightarrow r^2 = 5 \Rightarrow \pi r^2 = 5\pi$

13. Solution: 21 cm^2.

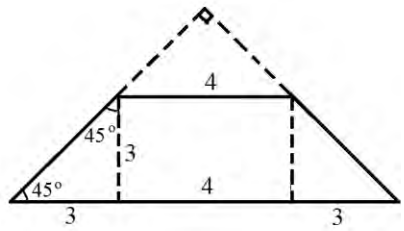


Method 1:
We drop the heights of the trapezoid as shown in the figure. So $AG = CG = 5$. $DE = CE = 4/2 = 2$. So $EG = 3$.

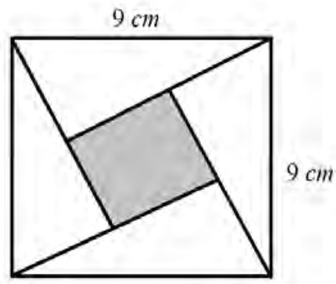
The area of the trapezoid is $\frac{(4+10)\times 3}{2} = 21$ cm^2.

Method 2:

We drop two heights of the trapezoid as shown in the figure. So the area is $\frac{(4+10)\times 3}{2} = 21\ \text{cm}^2$.



14. Solution: 64.8 cm^2.
Method 1:
We cut the rectangle along the diagonals and put them together to form a new figure as shown. Since the length is twice the width, the area of the square in the center is the same as the area of one small triangle. So the total area is $9 \times 9 \div (4 + 1) \times 4 = 64.8\ \text{cm}^2$.



15. Solution: 12π.

$$\pi \times \frac{6}{2} \times 4 = 12\pi$$

16. Solution: 47.
$9 = 9 \times 1 \times 1 = 3 \times 3 \times 1$.
When the rectangular prism is $9 \times 1 \times 1$, the surface area is $(9 \times 1 + 1 \times 1 + 1 \times 9) \times 2 = 38\ \text{cm}^2$ and the volume is $9 \times 1 \times 1 = 9\ \text{cm}^3$.
$38 + 9 = 47$.
When the rectangular prism is $3 \times 3 \times 3$, the surface area is $(3 \times 3 + 3 \times 1 + 1 \times 3) \times 2 = 30\ \text{cm}^2$ and the volume is $3 \times 3 \times 3 = 27\ \text{cm}^3$.
$30 + 9 = 39$
The answer is 47.

17. Solution: 10%.

Let x be the same reductions.

$50\times(1-\frac{x}{100})\times(1-\frac{x}{100})=40.5 \quad \Rightarrow \quad (1-\frac{x}{100})^2=0.81 \quad \Rightarrow 1-\frac{x}{100}=0.9 \quad \Rightarrow$

$\frac{x}{100}=0.1 \quad \Rightarrow \quad x=10\%$.

18. Solution: 300.

Alex finished $30\%-\frac{1}{5}=10\%$, or 30 more problems in second week than the number of problems he finished in the first week.

So the number of problems for his homework is 30 ÷ 10% = 300.

19. Solution: 1000 g.

The salt before adding water: 1000 × 25% = 250 g.

Let x be the salt added in grams.

$\frac{250+x}{1000+x}\times 100\%=25\%\times 2 \quad \Rightarrow \quad x = 500$ g.

In order to get a 30% solution, he needs to add

(250 + 500) ÷ 30% − (1000 + 500) = 1000 grams of water.

20. Solution: 24.

Let x be the number of girls. The the number of boys is $(40 - x)$.

We know that the average score for girls is 78, so we have

$80 \times 40 - 83 \times (40 - x) = 78x \quad \Rightarrow \quad x = 24$.

21. Solution: 30.

Let x be the age of Mr. Smith's son now. Mr. Smith's father's age is $15x$. Mr. Smith's age is $\frac{15}{2}x$.

We know that two years from now, the sum of three people's ages will be 100.

So we have $x+\frac{15}{2}x+15x+2\times 3=100 \quad \Rightarrow \quad x=4$.

$\frac{15}{2}x=30$.

22. Solution: 4 kg.
Method 1:
The sum of the weights of the boxes is (56 + 59 + 60)÷ 2 = 87.5
So the weights of them are:
87.5 – 56 = 31.5
87.5 – 59 = 28.5
87.5 – 60 = 27.5
The difference is 31.5 – 27.5 = 4 kg.

Method 2:
The difference in *kg* between heaviest box and the lightest box is 60 – 56 = 4 kg.

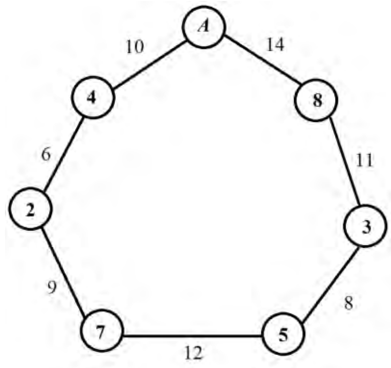
23. Solution: 40.
Let the number of students in class A be x. Then the number of students in class B is (80 – x).

$$\frac{3}{8}x+\frac{3}{5}(85-x)=42 \quad \Rightarrow \quad x=40.$$

24. Solution: 1500 kg.
Let a in kg be the weight of the orange originally.
First day the store sold $0.4a$ kg
Second day the store sold $0.6a \times 0.5 = 0.3a$ kg
Third day the store sold $0.4a \times 2/3 = 4a/15$ kg.
Then the orange left is $a-0.4a-0.3a-\frac{4}{15}a=\frac{a}{30}$ kg.
We know that $\frac{a}{30}=50 \quad \Rightarrow \quad a = 1500$ kg.

25. Solution: 18.
Let x be the number of students in both clubs.
$8 + 39 + 31 - x = 60 \quad \Rightarrow \quad x = 18.$

26. Solution:
Method 1:
Starting from A anticlockwise, the numbers in each circle should be:
$10 - A$
$6 - (10 - A) = A - 4$
$9 - (A - 4) = 13 - A$
$12 - (13 - A) = A - 1$
$8 - (A - 1) = 9 - A$
$11 - (9 - A) = 2 + A$
$A = 14 - (2 + A) = 12 - A$.
So $A = 6$.



Method 2:
We know that $14 + 11 + 8 + 12 + 9 + 6 + 10 = 70$.
So the sum of the seven consecutive positive integers is $70 \div 2 = 35$.
The middle number is $35 \div 7 = 5$.
So the seven numbers are 2, 3, 4, 5, 6, 7, and 8.
We see that only $6 + 8 = 14$.
So A =6 or 8.
We check and we see that $A = 6$.

27. Solution: 17/118.
$1 + 5 = 6$; $1 + 8 = 9$; $1 + 9 = 10$. LCM (6, 9, 10) = 90.
Since the volumes of three cups are the same, we assume that the volume is 90.
1:5, 1:8, and 1:9 $\Rightarrow$ 15:75, 10:80, and 9:81
The ratio of sugar to water of the mixture: $(15 + 10 + 9):(75 + 80 + 81) = 34: 236 = 17:118$.

28. Solution: 13.
We know that $1 + 2 = 3$ and $1 + 2 + 3 + 4 + 5 = 15$.
So at most we can get $15 - 3 + 1 = 13$ different sums.
We checked and only 1 and 2 are not achievable.

29. Solution: 16/33.

Method 1:
Since we have 6 pairs,
(A B C D E F)
(A_1 B_1 C_1 D_1 E_1 F_1)
we can select any pair (for example AA_1). So we have $\binom{6}{1}$ ways to choose any pair.

Now we have one pair and we need to select two shoes that are not paired.
(B C D E F)
(B_1 C_1 D_1 E_1 F_1)
We can select any two shoes from (BCDEF) or (B_1 C_1 D_1 E_1 F_1), which gives $\binom{5}{2}\times 2$ ways, or we can select any one shoe (for example, B) from (BCDEF) and the next one from (C_1 D_1 E_1 F_1), which gives $\binom{5}{1}\times\binom{4}{1}$ ways. $\dfrac{\binom{6}{1}\times[\binom{5}{2}\times 2+\binom{5}{1}\binom{4}{1}]}{\binom{12}{4}}=\dfrac{16}{33}$.

Method 2:
Since we have 6 pairs, we have $\binom{6}{1}$ ways to choose any pair. Now we have one pair and we need to select two shoes that are not paired. We can select any two shoes from the 10 shoes left. Then we take away 5 pairs to get the answer: $\dfrac{\binom{6}{1}\times[\binom{10}{2}-\binom{5}{1}]}{\binom{12}{4}}=\dfrac{16}{33}$.

30. Solution: 1508.
Let x be any number from 1 to 2011. m and n be two integers with $m > n$.
$x = m^2 - n^2 = (m-n)(m+n)$
We know that $m - n$ and $m + n$ have the same parity. If $m + n$ is even, $(m-n)(m+n)$ is a multiple of 4. If $m + n$ is odd, $(m-n)(m+n)$ is odd and x is odd.
So x is any number that is either a multiple of 4 or odd. There are 1006 odd numbers and 502 numbers that are multiples of 4. The answer is 1006 + 502 = 1508.

Solutions to Practice Test 8

1. Solution: 2.

$(196 + 1) \times 198 - 196 \times (198 + 1) = 196 \times 198 + 198 - 196 \times 198 - 196 = 198 - 196 = 2.$

2. Solution: 8.

13, 31, 17, 71, 37, 73, 79, 97.

3. Solution: 300.

Let the number of marbles in each box be x. The total number of marbles taken is $200 \times 6 = 1200$.

$6x - 1200 = 2 \times x \quad \Rightarrow \quad 4x = 1200 \quad \Rightarrow \quad x = 300.$

4. Solution: 2011.

$4 \times (n + 9) = 8080 \quad \Rightarrow \quad 4n = 8044 \quad \Rightarrow \quad n = 2011.$

5. Solution: 30.

The number of people is $14 + 16 + 1 = 31$. The number of students is $31 - 1 = 30$.

6. Solution: 8.

Let the number be x.

$8x - 13 = 6x + 3 \quad \Rightarrow \quad 2x = 16 \quad \Rightarrow \quad x = 8.$

7. Solution: 11.

The sum of the numbers is greater than 265.

We see that $1 + 2 + 3 + \dots + 22 = (1 + 22) \times 22 \div 2 = 253 < 265$.

$1 + 2 + 3 + \dots + 22 + 23 = (1 + 23) \times 23 \div 2 = 276 > 265$. This number works.

So $276 - 265 = 11$.

8. Solution: 987.

When 3 is subtracted from the number, the result is divisible by 4, 6, and 8 or divisible by 24. The greatest 3-digit number divisible by 24 is 984. So the answer is $984 + 3 = 987$.

9. Solution: 149.
When a positive integer is divided by 3, the remainder can only be 0, 1, or 2.
The remainders of all counting numbers 1, 2, 3, 4, 5, 6…, when divided by 3, are 1, 2, 0, 1, 2, 0, ….
When the remainder is 0, the number is divisible by 3.
So the 100^{th} positive integer that is not divisible by 3 is $100 \div 2 \times 3 - 1 = 149$.

10. Solution: 11.
When one number is added, we get 11 numbers. The sum of these 11 numbers is $11 \times 6 = 66$.
The number added is $66 - (1 + 2 + 3 + \ldots + 9 + 10) = 11$.

11. Solution: 27.
Let the sum of 8 numbers (after the greatest and the smallest numbers are removed) is a.
The smallest number of the original 10 numbers is $n = 22 \times 9 - a$.
The greatest number of the original 10 numbers is $m = 25 \times 9 - a$.
So the difference is $m - n = 25 \times 9 - a - (22 \times 9 - a) = 27$.

12. Solution: 9.
$30 - n = 15 + 12 - 6 \quad \Rightarrow \quad n = 9$
There are $30 - n = 30 - 9 = 21$ students are in the clubs. So $21 - 6 = 15$ students in one club only. Thus $m + n = 15 + 9 = 24$.

13. Solution: \$58.
Method 1:
Let x be the number of students.
Then price of the books is $(9x - 5) = (7x + 9) \quad \Rightarrow \quad x = 7$.
The cost of the books is $9x - 5 = 9 \times 7 - 5 = \$58$.

Method 2:
Each student pays $9 - 7 = \$2$more. Total they pay $5 + 9 = \$14$ more. So there are $14 \div 2 = 7$ students.
The cost of the books is $9 \times 7 - 5 = \$58$.

14. Solution: 0 meter.
When Alex runs 100 – 32 = 68 meters, Sam runs 100 – 15 = 80 meters.
When Alex runs 100 – 20= 80 meters, Sam runs $80 \times (85 \div 68) = 100$ meters.
So the answer is 0.

15. Solution: $390.
Method 1:
Let x be the money Alex invests. Bob at the beginning invests $(500 - x)$.
$x = [(500 - x) + 20] \times 3 \quad \Rightarrow \quad x = \$390$.

Method 2:
The total invest after Bob added $20 is $520. Let x be the money Alex invests and y be the money Bob invests. Then we have

$x + y = 520$ (1)

$x = 3y \quad \Rightarrow \quad y = x/3$ (2)

Substituting (2) into (1): $x + \dfrac{x}{3} = 520 \quad \Rightarrow \quad x = \$390$.

16. Solution: 2:1.
Let x be the speed of the ship going with the current and y be the speed of the ship going against the current.

$$\frac{21}{x} + \frac{4}{y} = \frac{12}{x} + \frac{7}{y} \quad \Rightarrow \quad \frac{9}{x} = \frac{3}{y} \quad \Rightarrow \quad \frac{x}{y} = \frac{3}{1} = \frac{2+1}{2-1}.$$

So the ratio of the speed of the ship to the speed of the current is 2:1.

Method 2:
Let Vs be the speed of the ship and Vc be the speed of the current and y be the speed of the ship going against the current.

$$\frac{21}{V_S + V_C} + \frac{4}{V_S - V_C} = \frac{12}{V_S + V_C} + \frac{7}{V_S - V_C} \quad \Rightarrow \quad \frac{3}{V_S + V_C} = \frac{1}{V_S - V_C} \quad \Rightarrow$$

$$3(V_S - V_C) = V_S + V_C \quad \Rightarrow \quad \frac{V_S}{V_C} = \frac{2}{1}$$

17. Solution: $35.
Method 1:

The cost of 5 desks is $10 \times 5 = \$50$ more than 5 chairs.
The cost of 5 desks and 8 chairs is \$50 more than 13 chairs.
So with \$(375 – 50) we can buy 13 chairs. So each chair costs $\$(375 - 50) \div 13 = \$25$.
Each desk costs $25 + 10 = \$35$.
Method 2:
Let x be the cost of each desk and y be the cost of each chair.

$$5x + 8y = 375 \qquad (1)$$

$$x - 10 = y \qquad (2)$$

Substituting (2) into (1): $5x + 8(x - 10) = 375 \quad \Rightarrow \quad x = \$35$.

18. Solution: 47.
Let x be the seconds needed for Alex to run one round.

$$6 \times \frac{x}{2} + 4 \times \frac{x}{2} = 400 \quad \Rightarrow \quad x = 80.$$

The time for Alex to run the first half of the distance is $\frac{200}{6} = \frac{100}{3}$ seconds and the time for Alex to run the second half of the distance is $80 - \frac{100}{3} = \frac{140}{3} \approx 47$ seconds

19. Solution: 32°.
Method 1:
At 4: 00 P.M, the angel between the hour hand and the minute hand is $\frac{360}{12} \times 4 = 120°$.

In 16 minutes, the minute hand travels $\frac{360}{60} \times 6 = 96°$ and the hour hand travels $\frac{360}{12 \times 60} = 8°$. The answer is $120° + 8° - 96° = 32°$.

Method 2:
At 4: 00 P.M, the angel between the hour hand and the minute hand is $\frac{360}{12} \times 4 = 120°$.

In 16 minutes, the minute hand travels at a relative speed of $\frac{11°}{2}$ per minutes if the hours hand does not move. So the minute hand travels $\frac{11}{2} \times 16 = 88°$. The answer is 120° – 88° = 32°.

20. Solution: 5.
Let S be the overlapping area of two squares.
The non-overlapping area is $(5 \times 5 - S) - (4 \times 4 - S) = 5 \times 5 - 4 \times 4 = 9$ cm^2.

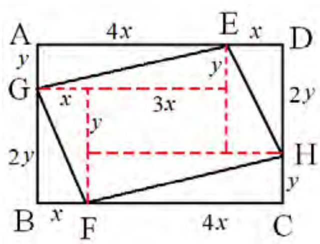
21. Solution: 80.

$$\angle A + \angle B + \angle C = 180° \qquad (1)$$

$$130° = \angle A + \frac{\angle B}{2} + \frac{\angle C}{2} \quad \Rightarrow \quad 2\angle A + \angle B + \angle C = 260° \qquad (2)$$

(2) – (1): $\angle A = 80°$.

22. Solution: 35.
We divide the parallelogram $EGFH$ into five parts as shown in the figure.
The area of the rectangle $ABCD$ is $5x \times 3y = 15xy$.



The area of $EGFH$ is $\frac{4x \times y}{2} \times 2 + \frac{x \times 2y}{2} \times 2 + 3x \times y = 21$

$$\Rightarrow \quad 9xy = 21 \quad \Rightarrow \quad xy = \frac{7}{3}.$$

So $15xy = 15 \times \frac{7}{3} = 35$.

23. Solution: 5.
The last digit must be 0 or 2.
When the last digit is 0, The tens digit can be 1, 2, or 3.
When the last digit is 2, The tens digit can be 1, or 3. So there are 5 different 2-digit even numbers.

24. Solution: 14.

Since $\overline{a2011b}$ is divisible by 45, it is divisible by both 5 and 9. The last digit must be 0 or 5.

$a + 2 + 0 + 1 + 1 + b = a + 4 + b$.

When $b = 5$, $a + 4 + b = a + 4 + 5 = a + 9$. So a must be 9.

When $b = 50$, $a + 4 + b = a + 4 + 0 = a + 4$. So a must be 5.

So the sum is 9 + 5 = 14.

25. Solution: 3122.

Let the 7 3-digit numbers be *abc, bcd, cde, def, efg, fgh, ghi.*

The sum of these 3-digit numbers is $100a+110b+111(c+d+e+f+g)+11h+i$.

Since we want the smallest sum, we let $i = 9$, $h = 8$, $a = 7$, $b = 6$, c, d, e, f, and g can be1 to 5.

So the smallest sum is 3122.

26. Solution: 30.

Let d, c, s be the number of dragonfly, cicada, and spider, respectively.

$d + c + s = 60$ (1)

$6d + 6c + 8s = 400$ (2)

$2d + c = 50$ (3)

(1) + (2) + (3): $9d + 8c + 9s = 510$ (4)

(1) × 9 – (4): $c = 30$.

Substituting $c = 30$ into (3): $2d + 30 = 50 \quad \Rightarrow \quad d = 10$.

27. Solution: 1.

We have the following pairs: (1, 2); (1, 3); (1, 4); (2, 3); (2, 4); (3, 4).

The differences are 1, 2, 3, 1, 2, and 1. We see 3' 1's so the most possible value of x is 1.

28. Solution: 75 pages.

Let a be the number of pages Alex read the first day, b be the number of pages Alex read the second day.

Then the number of pages he read the third, fourth, fifth, and sixth day, respectively, will be $(a + b)$, $(a + 2b)$, $(2a + 3b)$, $(3a + 5b)$.

$(a + b) + (a + 2b) + (2a + 3b) + (3a + 5b) = 8a + 12b = 4(2a + 3b) = 300$.
So $2a + 3b = 75$. This is the number of page Alex read in fifth day.

29. Solution: 8.
We divide the regular hexagon *ABCDEF* into 12 congruent triangles as shown. The area of rhombus *MPNO* is $24 \div 12 \times 4 = 8$.

30. Solution: 796.
We see that 863 and 483 have the same units digit. So ♀ is 3.
So *C* is 375. ※ represents 7. *A* is 796.

Solutions to Practice Test 9

1. Solution: 2011.

$[223 \times 1.25 + 22.3 \times 75 + 2.33 \times 125] \times 0.9 + 4 = 223\times [1.25 + 7.5 + 1.25] \times 0.9 + 4$

$= 223\times 10 \times \dfrac{9}{10} + 4 = 2007 + 4 = 2011.$

2. Solution: $100\dfrac{1023}{1024}$.

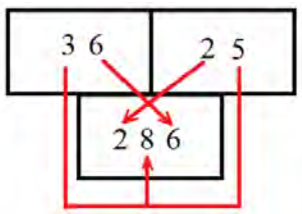
$$1+3+5+7+9+11+13+15+17+19+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\frac{1}{128}+\frac{1}{256}+\frac{1}{512}+\frac{1}{1024}$$

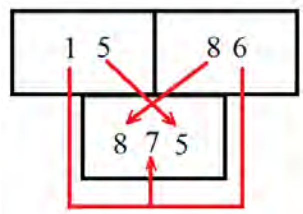
$$= 10^2+\frac{512+256+128+64+32+16+8+4+2+1}{1024}=100\frac{1023}{1024}$$

3. Solution: 19.

$$S=\frac{7}{20}(1+2+3+\cdots+10)=\frac{7}{20}\times 55=19.25\approx 19$$

4. Solution:





5. Solution: 13.

Consider the worst case scenario, taking out 10 gloves of the same color in 10 times. At least 3 more gloves need to be taken out from the rest of gloves of two colors to have another pair with the same color.

6. Solution: 439.

Method 1:

Let the hundred digit be x, the units digit be $x + 5$, and the tens digit be $16 – x – (x + 5) = 11 – 2x$.

$[100x + (11 - 2x) \times 10 + (x + 5)] \times 2 = (x + 5) \times 100 + (11 - 2x) \times 10 + x - 56.$

$162x + 230 = 81x + 554 \quad \Rightarrow \quad 81x = 324 \quad \Rightarrow \quad x = 4.$

$x + 5 = 9$ and $11 - 2x = 11 - 8 = 3$. The original number is 439.

Method 2:

Let the 3-digit number be *abc* and the new number be *cba.*

$a + b + c = 16 \qquad (1)$

$2(100a + 10b + c) = 100c + 10b + c - 56 \Rightarrow \quad -199a - 10b + 98c = 56 \qquad (2)$

(1) × 10 + (2): $-198a + 108c = 216 \quad \Rightarrow \quad 4(c - 2) = 7a$.

Since 4 and 7 are relatively prime and a and c are digits, so $a = 4$ and $c = 9$.
Substituting these values into (1), $b = 3$. The original number is 439.

7. Solution: 6.

Since $150 = 2 \times 3 \times 5^2$, the side of any rectangle needs to be a multiple of 2. 150 has 2 × 2 × 3 = 12 factors. So there are 12/2 = 6 ways to form a rectangle.

8. Solution: 8 pairs.

(3, 5); (5, 7); (11, 13); (17, 19); (29, 31); (41, 43); (59, 61); (71, 73).

9. Solution: 13.

Since 361361 = 361 × 1001 = 361 × 7 × 11 × 13, their ages are 7, 11, and 13. Alex is the oldest so he is 13.

10. Solution: 20.

1995 = 3 × 5 × 7 × 19 = 21 × 95 = 35 × 57.

We have two values and 20 is the greater one:

2 + 1 + 9 + 5 = 17; 3 + 5 + 5 + 7 = 20.

11. Solution: 56.

Let the two numbers be x and y.

$xy = 7(x + y) \Rightarrow \quad xy - 7x - 7y = 0 \quad \Rightarrow \quad xy - 7x - 7y + 49 = 49 \Rightarrow$

$(x - 7)(y - 7) = 1 \times 49 = 7 \times 7$

So we have two cases:

(1) $x - 7 = 49$ and $y - 7 = 1$.

We get $x = 56$ and $y = 8$.

(2) $x-7=7$ and $y-7=7$.

We get $x = 14$ and $y = 8$.

Since x and y are different, the answer is 56.

12. Solution: 84.

Let x number of sixth graders.

The number of boys is $\frac{4}{9}x$ and the number of girls is $1-\frac{4}{9}x$.

The scores of boys is $90 \times \frac{4}{9}x$. The scores of boys is $90 \times \frac{4}{9}x\times\frac{7}{6}$.

The average score for girls is $\dfrac{90\times\frac{4}{9}x\times\frac{7}{6}}{1-\frac{4}{9}x}=84$.

13. Solution: 3000 m.

Let the distance one way be *x,* the time used to climb for Alex be *t.*

Alex's speed is *x/t* meters per hour and Bob' s speed is (x – 500)/t meters per hour.

$$\frac{500}{\frac{x-500}{t}}+\frac{\frac{x}{2}}{\frac{2(x-500)}{t}}=\frac{t}{2} \quad\Rightarrow\quad \frac{500}{x-500}+\frac{x}{4(x-500)}=\frac{1}{2}\Rightarrow \quad x=3000.$$

14. Solution: $4:54\frac{6}{11}$.

Let x be the minutes needed for the hour hand and the minute hand to be in the opposite position.

The minute hand travels 6° in a minute and the hour hand travels 0.5° in a minute.

$$6x-0.5x=180°+120° \quad\Rightarrow\quad x=\frac{600}{11}=54\frac{6}{11}.$$

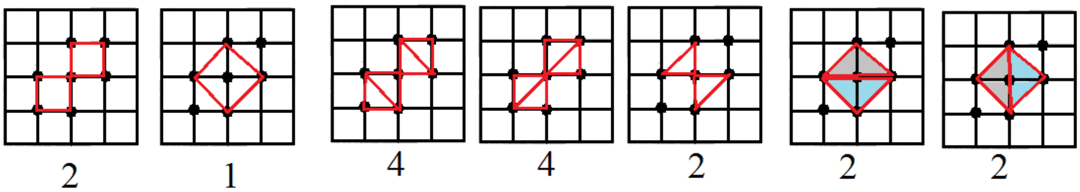
So the time is $4:54\frac{6}{11}$.

15. Solution: 4.

If $\overline{k45k7}$ is divisible by 3, $k + k + 4 + 5 + 7 = 2k + 16$ must be divisible by 3. So k can be 1, 4, or 7. So $m = 3$.

If $\overline{k45k7}$ is divisible by 9, $k + k + 4 + 5 + 7 = 2k + 16$ must be divisible by 9. So k can only be 1. So $n = 3$.
The answer is $m + n = 3 + 1 = 4$.

16. Solution: 17.
$m = 3$ and $n = 14$. $m + n = 3 + 14 = 17$.



17. Solution: 36.
Let the number of girls be g and the number of boys be b.

$$5g = 4b \quad \Rightarrow \quad b = \frac{5}{4}g \qquad (1)$$

$$(g - 10) \times 2 = (b - 30) \times 3 + 7 \qquad (2)$$

Substituting (1) into (2): $g = 36$.

18. Solution: 662.
Method 1:

2 9 17 24 32 39 47 54 62

7 8 7 8 7 8 7 8

We see that pattern $32 = 2 + 7 \times 2 + 8 \times 2$, $39 = 2 + 7 \times 3 + 8 \times 3$.

$$a_n = 2 + 7 \times \frac{n}{2} + 8 \times \frac{n-2}{2}, \text{ if } n \text{ is even}$$

$$a_n = 2 + 7 \times \frac{n-1}{2} + 8 \times \frac{n-1}{2}, \text{ if } n \text{ is odd.}$$

$$a_{89} = 2 + 7 \times \frac{89-1}{2} + 8 \times \frac{89-1}{2} = 2 + 7 \times 44 + 8 \times 44 = 662.$$

Method 2:
The sequence has two common differences: 7 and 8.

The general term is $a_n = a_1 + \dfrac{(n-1)(d_1+d_2)}{2} + \dfrac{1+(-1)^n}{2} \times \dfrac{d_1 - d_2}{2}$ or

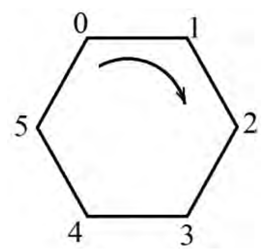
$a_n = a_1 + \left\lfloor \dfrac{n}{2} \right\rfloor d_1 + \left\lfloor \dfrac{n-1}{2} \right\rfloor \times d_2$.

$a_{89} = 2 + 7 \times \left\lfloor \dfrac{89}{2} \right\rfloor + 8 \times \left\lfloor \dfrac{89-1}{2} \right\rfloor = 2 + 7 \times 44 + 8 \times 44 = 662$.

19. Solution: 576.
Step 1: assign 4 boys to four schools with 4! = 24 ways.
Step 2: assign 4 girls to four schools with 4! = 24 ways.
By the fundamental counting principle, The answer is 24 × 24 = 576.

20. Solution: 2.

Note that $1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$.



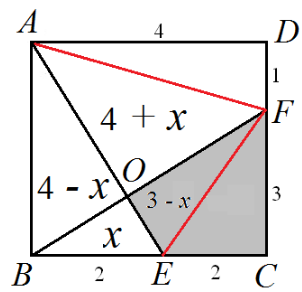
These triangular numbers are 1, 3, 6, 10,….
When divided by 6, the remainders of these numbers can only be 0, 1, 3, and 4 (not 2 or 5). So positions 2 and 5 will not be landed.

21. Solution:
Connect *AF*, *EF*.

$S_{\Delta ABF} = 8$. $S_{\Delta BEF} = 3$. $S_{\Delta BCF} = 6$. $S_{\Delta ABE} = 4$.

Let $S_{\Delta BOE} = x$. Then $S_{\Delta ABO} = 4 - x$.

$S_{\Delta ADF} = 8 - (4 - x) = 4 + x$. $S_{\Delta EOF} = 3 - x$



$\dfrac{4-x}{x} = \dfrac{4+x}{3-x} \Rightarrow \quad x = \dfrac{12}{11}$.

So the shaded area is $3 - x + \dfrac{2 \times 3}{2} = 3 - \dfrac{12}{11} + 3 = 4\dfrac{10}{11}$.

22. Solution: 130 km.
Method 1:
Let the distance from Kinston to Durham be d, and their speeds be V_a, V_b, respectively.

$$\frac{60}{V_B}=\frac{d-60}{V_A} \qquad (1)$$

$$\frac{d-60+50}{V_B}=\frac{60+60+d-60-50}{V_A} \qquad (1)$$

(2) ÷ (1): $\frac{d-10}{60}=\frac{d+10}{d-60} \quad \Rightarrow \quad \frac{d-10}{60}=\frac{20}{d-120} \quad \Rightarrow \quad d = 130$ km.

Method 2:
When they met the first time, they covered d, the distance from Kinston to Durham.
When they met the second time, they covered $2d$.

When they met the first time, car B travelled 60 km.
When they met the second time, car B travelled 120 km. So $d - 60 + 50 = 120 \quad \Rightarrow$
$d = 130$.

23. Solution: 119.
One-digit palindrome numbers: 1, 2, to 9. There are 9 of them.
Two-digit palindrome numbers: 11, 22. to 99. There are 9 of them.
Three-digit palindrome numbers: 101, 202, 222, to 999. There are 90 of them.
Four-digit palindrome numbers: 1001, 1111, 1221, 1331, 1441, 1551, 1661, 1771, 1881, 1991, 2002. There are 11 of them.

24. Solution: 24.

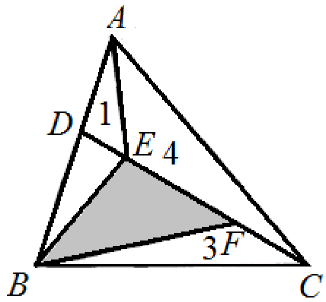
$$S=(20-5)+(200-5)+\cdots+(\underbrace{20\cdots0}_{45\text{zeros}}-5)=20+200+\cdots+\underbrace{20\cdots0}_{45\text{zeros}}-5\times45$$

$$=\underbrace{22\cdots2}_{45\ \ 2\text{'s}}0-225=\underbrace{22\cdots2}_{42\ \ 2\text{'s}}1995\,.$$

The sum of the last four digits is 1 + 9 + 9 + 5 = 24.
25. Solution: 2.
We see that 1, 8, 27, 64, 125, 216, … are the cubic numbers.
So we have $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, $6^3 = 216$, ….

Since 10^3 to 21^3 are all 4-digit numbers, we have $12 \times 4 = 48$ digits.
Now we have $100 - 21 - 48 = 31$ digits left to count.
Since 22^3 to 27^3 are all 5-digit numbers, we have $5 \times 6 = 30$ digits.
So the 100th digit is the first digit in the number $28^3 = 21952$, which is 2.

26. Solution: 5.



$S_{\Delta ADC} = 1 + 4 = 5$

$S_{\Delta BDC} = 7 + 3 = 10$

So $\dfrac{S_{\Delta ADE}}{S_{\Delta BDE}} = \dfrac{AD}{BD} = \dfrac{S_{\Delta ADC}}{S_{\Delta BDC}} = \dfrac{5}{10} = \dfrac{1}{2}$.

So $S_{\Delta BDE} = 2$ and $S_{\Delta BEF} = 7 - 2 = 5$.

27. Solution: 6.

Let $3^{2011} = (n+1) + (n+2) + \cdots + (n+k)$.

So $3^{2011} = kn + \dfrac{k(k+1)}{2} \quad \Rightarrow \quad 2 \times 3^{2011} = k(2n + k + 1)$.

We know that $k < 2n + k + 1$ and they have the opposite parity.

$2 \times 3^{2011} = 2 \times 3^{1005} \times 3^{1006}$.

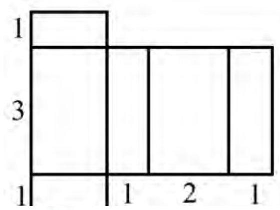
So the greatest value of k is 2×3^{1005}. The lat digit of k is $2 \times 3 \times (3^2)^{502} \equiv 6 \mod 10$.

28. Solution: 1.5 hours.

Now they are 15 miles apart. Let t be the time when they 30 miles apart.

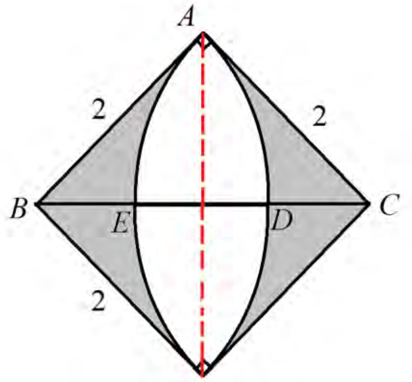
$(50 - 40) \times t = 15 \quad \Rightarrow \quad t = 1.5$ hours.

29. Solution: 22 cm.



When we unfold the cube, we need to cut seven edges and keep 5 edges. In order to get the smallest value of the perimeter, we need to cut as many shorter edges as we can. So we cut the 4 edges of 1 cm, 2 edges of 2 cm, and one edge of 3 cm (we need cut 3 cm edge at least one). So the smallest value of the perimeter is $(3 \times 1 + 2 \times 2 + 1 \times 4) \times 2 = 22$ cm.

30. Solution: $4 - \pi$.



Unshaded area is $\frac{1}{4}\pi \times 2^2 - \frac{1}{2} \times 2^2 = \pi - 2$.

The shaded area is $\frac{1}{2} \times 2^2 - (\pi - 2) = 4 - \pi$.

Solutions to Practice Test 10

1. Solution: 58.

$$2011 \div 37 + 270 \div (37 \times 2) = 2011 \div 37 + 135 \div 37 = (2011 + 135) \div 37 = 58$$

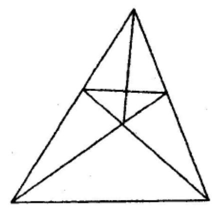
2. Solution:

$$82.54 + 835.27 - 20.38 \div 2 + 2 \times 6.23 - 390.81 - 9 \times 1.03$$
$$= 82.54 + 835.27 - 10.19 + 12.46 - 390.81 - 9.27$$
$$= (82.54 + 12.46) + (835.27 - 9.27) - (10.19 + 390.81)$$
$$= 95 + 826 - 401 = 921 - 401 = 520$$

3. Solution: 154 cm.

The sum of the height of two girls and one boy is $150 \times 2 + 162 = 462$.
The average is 462/3 = 154 cm. This represents the average height of all students.

4. Solution: 20.



Count by 1: 7.
Count by 2: 6.
Count by 3: 4.
Count by 4: 0.
Count by 5: 2. Count by 6: 0.
Count by 7: 1:
Total 7 + 6 + 4 + 2 + 1 = 20.

5. Solution: 48.

The number of fruits each student gets is 1 + 3 + 5 = 9 which is three times of the number of apples each student gets. So the total number of fruits left is three times of the number of apples left. So total number of fruits left is $24 \times 3 = 72$ and the sum of the numbers of oranges and bananas left is 72 – 24 = 48.

6. Solution:

The pattern repeats every 9 balls: RBBYBBYBB. There are 2 yellow balls every nine balls.
$2011 \div 9 = 223\ r\ 4$.

The last four balls are RBBY.
We have $223 \times 2 + 1 = 447$ yellow balls.

7. Solution: 14.

$6 = 5 + 1 + 0$	4 (510, 501, 150, 105)
$= 4 + 2 + 0$	4 (420, 402, 240, 204)
$= 3 + 2 + 1$	6 (321. 312. 231, 213, 123, 132)

8. Solution: 3.
Let a, b, and c be the weights of the boxes with $a \le b \le c$.

$a + b = 63$ (1)
$a + c = 65$ (2)
$b + c = 66$ (3)

(1) + (2) + (3): $a + b + c = 97$ (4)
(4) – (3): $a = 31$
(4) – (1): $c = 34$
The answer is $34 – 31 = 3$.

9. Solution: 25.
Let a, b, c, and d represent the number of coins valued at 1¢, 2¢, 5¢, and 10¢, respectively.

$a + 2b + 5c + 10d = 16$ (1)

We see that at most $d = 1$.

When $d = 1$, (1) becomes: $a + 2b + 5c = 6$ (2)

In (2), c is at most 1. When $c = 1$, (2) becomes: $a + 2b = 1$ (3)
So $a = 1$ and $b = 0$.
We get one solution: (1, 0, 1, 1).

In (2), if c is 0, (2) becomes: $a + 2b = 6$ (4)
We get 4 solutions: (6, 0, 0, 1), (4, 1, 0, 1), (2, 2, 0, 1), (0, 3, 0, 1).

When $d = 0$, (1) becomes: $a + 2b + 5c = 16$ (5)

In (5), c is at most 3.
When $c = 3$, (5) becomes: $a + 2b = 1$
We get 1 solution: (1, 0, 3, 0).

When $c = 2$, (5) becomes: $a + 2b = 6$
We get 4 solutions: (6, 0, 2, 0), (4, 1, 2, 0), (2, 2, 2, 0), (0, 3, 2, 0).

When $c = 1$, (5) becomes: $a + 2b = 11$
We get 6 solutions: (11, 0, 1, 0), (9, 1, 1, 0), (7, 2, 1, 0), 5, 3, 1, 0), (3, 4, 1, 0), (1, 5, 1, 0).

When $c = 0$, (5) becomes: $a + 2b = 16$.
We get 9 solutions: (16, 0, 0, 0), (14, 1, 1, 0), (12, 2, 0, 0), 10, 3, 0, 0), (8, 4, 0, 0), (6, 5, 0, 0), (4, 6, 0, 0), (2, 7, 0, 0), (0, 8, 0, 0).

Total we have 1 + 4 + 1 + 4 + 6 + 9 = 25 different ways.

10. Solution: 22412.
431 × 52=22412.

11. Solution: 9.
The last digit of each number is listed below:
1, 1, 2, 3, 5, 8, 3, 1, 4, 5,
9, 4, 3, 7, 0, 7, 7, 4, 1, 5,
6, 1, 7, 8, 5, 3, 8, 1, 9, 0,
9, 9, 8, 7, 5, 2, 7, 9, 6, 5,
1, 6, 7, 3, 0, 3, 3, 6, 8, 5,
4, 9, 3, 2, 5, 7, 2, 9, 1, 0,
1, 1,……

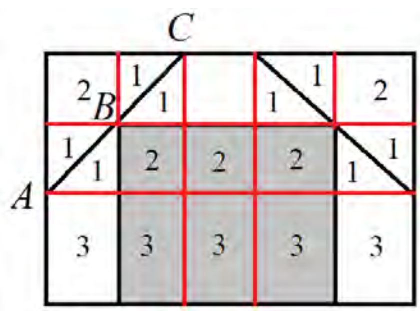
The pattern will repeat every 60 digits. $2011 = 60 \times 33 + 31$.
So 2011 digit is the same as the 31st digit 9.

12. Solution: 16.
When the digits are 5 and 4 for two black squares, we have $3! \times 2 = 12$ ways.
When the digits are 5 and 3 for two black squares, we have $2 \times 2 = 4$ ways.
The answer is $12 + 4 = 16$.

13. Solution: 60 minutes.
We see four odd nodes in the figure. So at least Alex needs to walk twice two hypotenuses.
The distance is $(204 + 240 + 100 + 100) \times 3 + 260(4 + 2) = 3600$ meters.
The time is $3600 \div 60 = 60$ minutes.

14. Solution: 14.
$420 = 2^2 \times 3 \times 5 \times 7 = 2 \times 210 = 3 \times 140 = 4 \times 105 = 5 \times 84 = 6 \times 70 = 7 \times 60 = 10 \times 42$
$= 12 \times 35 = 14 \times 30 = 15 \times 28 = 20 \times 21$.
We see that $14 \times 30 = (14 + 1)(30 - 2) = 15 \times 28$. So 14 is the answer.

15. Solution: 861 cm^2.
As shown in the figure, we divide the rectangle into smaller shapes. The shaded area 15 parts and the total area is 35 parts. So the shaded area is $2009 \times \frac{15}{35} = 861$.



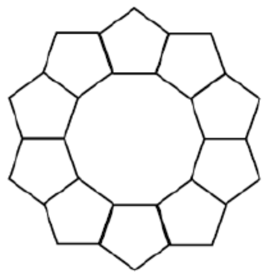
16. Solution: 1434.
The sum of the six digits is $4+ 5+ 6 + 7 + 8 + 9 = 39$, which is divisible by 3. We see that 667 and 3 are relatively prime. So this 6-digit number is divisible by $667 \times 3 = 2001$.
Let the 6-digit number be $\overline{abcdef}$. Then $\overline{abcdef} = 2001 \times m = (2000 + 1)m = 2000m + m$, where m is a factor of $\overline{abcdef}$. We see that if we separate $\overline{abcdef}$ into two 3-digit

numbers $\overline{abc}$ and $\overline{def}$, we have the following relationship: $2\overline{abc} = \overline{def}$. It is easy to see that 965 =2 × 478. So the quotient when 965478 is divided by 667 is 1434.

17. Solution: 19.
The sum could be 3, 6, 9, 12, 18, and 21.
3 = 1 + 2 2 ways.
6 = 5 + 1 = 4 + 2 = 3 + 2 + 1 4 ways
9 = 6 + 3 = 5 + 4 = 6 + 2 + 1 = 5 + 3 + 1 = 4 + 3 + 2 5 ways
12 = 6 + 5 + 1 = 6 + 4 + 2 = 6 + 3 + 2 + 1 = 5 + 4 + 3 = 5 + 4 + 2 + 1 5 ways
18 = 6 + 5 + 4 + 3 = 6 + 5 + 3 + 2 2 ways
21 = 6 + 5 + 4 + 3 + 2 + 1 1 way.
The answer is 2 + 4 + 5 + 5 + 2 + 1 = 19.

18. Solution: 670.
Let the number be N. $N + 2$ is divisible by 24, 28, and 32. So $N + 2$ is divisible by LCM (24, 28, 32) = 672.
Since N is less than 1000, $N + 2 = 672$. $N = 670$.

19. Solution: 10.
The interior angle of a regular pentagon is (5 – 2) × 180° ÷ 5 = 108°.
The sum of the exterior angles is 360°. Each exterior angle is 108° × 2 – 180° = 36°. So we need 360° ÷ 36° = 10 regular pentagons.



20. Solution: 75531.
We know that none of the five digits is zero. There are 2 5's. Let other three digits be $a \ge b \ge c$.
$a \times b \times c \times 5 \times 5 = 25 (a + b + c + 5 + 5) \quad \Rightarrow \quad a \times b \times c = a + b + c + 10 \quad (1)$
Case 1: if $a = 9$, $a + b + c + 10$ is at least 21 and at most 37. So $b \times c = 3$ or 4. (1) has no solution.

Case 2: if $a = 8$, $a + b + c + 10$ is at least 20 and at most 34. So $b \times c = 3$ or 4. (1) has no solution.

Case 3: if $a = 7$, $a + b + c + 10$ is at least 19 and at most 31. So $b \times c = 3$ or 4.

$b \times c = 3 \quad \Rightarrow \quad b = 3, c = 1$. We get the 5-digit number: 75531.

$b \times c = 4 \quad \Rightarrow \quad b + c = 11$. No solution.

Case 4: if $a = 6$, $a + b + c + 10$ is at least 18 and at most 28. So $b \times c = 3$ or 4. (1) has no solution.

Case 5: if $a = 5$, $a + b + c + 10$ is at least 17 and at most 25. So $b \times c = 4$ or 5.

$b \times c = 4 \quad \Rightarrow \quad b = 4, c = 1$; (we get the 5-digit number 55541)

Other values of b and c do not result any solution.

Case 6: if $a = 4$, $a + b + c + 10$ is at least 16 and at most 22. So $b \times c = 4$ or 5.

We have no solution.

Case 7: if $a = 3$, $a + b + c + 10$ is at least 15 and at most 19. So $b \times c = 5$ (no solution since b or $c > a$) or 6.

$b \times c = 6 \quad \Rightarrow \quad b = 3, c = 2$; (we get the 5-digit number 55332)

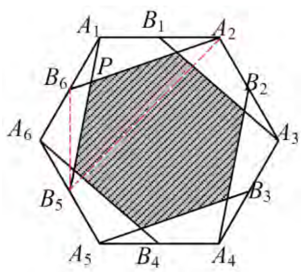
The greatest possible value of the 5-digit number is 75531.

Note: This is a problem in a 2009 math competition. The original official answer key was 55332.

21. Solution: 1148 cm^2.

Connect B_5, and B_6 , B_5 and A_2. Let the area of regular hexagon $A_1A_2A_3A_4A_5A_6$ be 1,

$$\frac{S_{\Delta A_1B_6B_5}}{S_{A_1A_2A_3A_4A_5A_6}} = \frac{1}{6} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{24} \Rightarrow \qquad S_{\Delta A_1B_6B_5} = \frac{1}{24}$$



$$S_{\Delta A_1A_2B_5} = \frac{1}{6} \times \frac{3}{2} = \frac{1}{4}.$$

$$\frac{B_6P}{A_2P} = \frac{S_{\Delta A_1B_6B_5}}{S_{\Delta A_1A_2B_5}} = \frac{\frac{1}{24}}{\frac{1}{4}} = \frac{1}{6}.$$

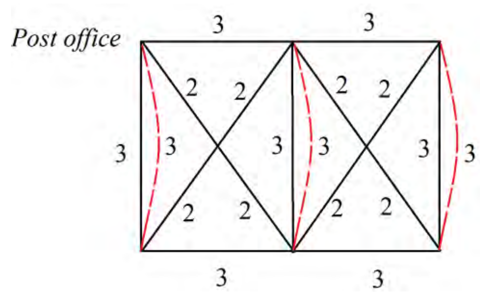
$$S_{\Delta A_1 A_2 P} = S_{\Delta A_1 A_2 B_6} \times \frac{6}{7} = \frac{1}{12} = \frac{1}{14}.$$

The shaded area is $2009 \times (1 - \frac{1}{14} \times 6) = 1148$ cm^2.

22. Solution: 37:11.
When we form the 4×4×4 cube, we put $(4 - 2)^3 = 8$ black unit cubes inside.
We have put $(4 - 2)^2 \times 6 = 24$ cubes on the faces but not on the edges. Among them, we use 22 black cubes.

Among the $4 \times 4 \times 6 = 96$ exterior cubes, 22 are black and $96 - 22 = 74$ are white. The ratio is 74:22 = 37:11.

23. Solution: 46 miles.
The total lengths of the street is $3 \times 7 + 2 \times 8 = 37$ miles. There are 6 odd nodes. We need to add 3 streets of length 3 miles in order to change the odd nodes to even. So the distance is $37 + 3 \times 3 = 46$ miles.



24. Solution: 22.
Let the first three teams be *A, B*, and *C* in the order of first place, second place, and third place.

They are winners with all 9 teams left. Between them, *A* beats *B*, *B* beats *C*, and *C* beats *A*. So everyone wins ten games with 30 points.

Each of the 9 teams ties with other 8 teams. So each of them has 8 points.
The difference is $30 - 8 = 22$.

25. Solution: 5.

$52\% = \frac{13}{25}$. So 25 is a factor of N^3. Thus 5 is a factor of N.

When $N = 5$, there are $5^3 = 125$ unit cubes. The number of cubes without any paint is $(5 - 2)^3 = 27$. The number of cubes with paint is $125 \times 52\% = 65$. $27 + 65 = 92 < 125$. It works.

When $N = 10$, there are $10^3 = 1000$ unit cubes. The number of cubes without any paint is $(10 - 2)^3 = 512$. The number of cubes with paint is $1000 \times 52\% = 520$.
$512 + 520 = 1032 > 1000$. It does not work. All other cases with $N > 10$ will not work. So $N = 5$.

26. Solution: 23.
$126 = 2 \times 3^2 \times 7 = 14 \times 9$.
Since these two numbers are relatively prime, they can be 14 and 9. So the sum is 23.

27. Solution: 65.
$\overline{AB}^2 - \overline{BA}^2 = (\overline{AB} - \overline{BA})(\overline{AB} + \overline{BA}) = (10A + B + 10B + A)(10A + B - 10B - A)$
$= 11(A + B) \times 9(A - B)$.
We see that 9 is a square number. So if $A + B = 11$ and $A - B = 1$, $11(A + B) \times 9(A - B)$ will be a square number.
So we have $A = 6$ and $B = 5$. So the answer is 65.

28. Solution: 651.

$$\frac{2n+1}{n\times(n+1)(n+2)} = \frac{n}{n\times(n+1)(n+2)} + \frac{n+1}{n\times(n+1)(n+2)} = \frac{1}{(n+1)(n+2)} + \frac{1}{n\times(n+2)}$$

$$1155\times(\frac{5}{2\times3\times4} + \frac{7}{3\times4\times5} + \cdots + \frac{17}{8\times9\times10} + \frac{19}{9\times10\times11})$$

$$= 1155\times(\frac{1}{2\times4} + \frac{1}{3\times4} + \frac{1}{3\times5} + \frac{1}{4\times5} + \cdots + \frac{1}{8\times10} + \frac{1}{9\times10} + \frac{1}{9\times11} + \frac{1}{10\times11})$$

$$= 1155\times[(\frac{1}{3\times4} + \frac{1}{4\times5} + \cdots + \frac{1}{10\times11}) + (\frac{1}{2\times4} + \frac{1}{3\times5} + \cdots \frac{1}{8\times10} + \frac{1}{9\times11})$$

$$= 1155\times[(\frac{1}{3} - \frac{1}{11}) + \frac{1}{2}(\frac{1}{2} + \frac{1}{3} - \frac{1}{10} - \frac{1}{11}) = 651$$

29. Solution: 30 minutes.

The hour hand moves one round while the minute hand moves six rounds. So the minute hand moves 5 more round than the hour hand every 60 minutes. Thus the minute hand needs 12 minutes to move one round more than the hour hand.

30. Solution: 27 cm^2.

$$S_{\Delta APD} + S_{\Delta BPC} = \frac{1}{2} S_{ABCD}.$$

$$S_{\Delta BPD} + S_{\Delta ABP} + S_{\Delta APD} = \frac{1}{2} S_{ABCD}.$$

$$S_{\Delta BPD} + S_{\Delta ABP} + S_{\Delta APD} = S_{\Delta APD} + S_{\Delta BPC}$$

$$S_{\Delta BPD} = S_{\Delta BPC} - S_{\Delta ABP} = 100 - 73 = 27 \text{ cm}^2.$$

Solutions to Practice Test 11

1. Solution: 2.
$$\frac{1}{2}+\frac{14}{28}+\frac{104}{208}+\frac{1004}{2008}=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=2$$

2. Solution: 4.
$1*2=1+2\div 1=2$
$3*3=3+3\div 3=4$

3. Solution: $20:15:12$.
$$A:B:C=\frac{5}{3}C:\frac{5}{4}C:C=\frac{5}{3}:4:1=(\frac{5}{3}\times 12):(\frac{5}{4}\times 12):(1\times 12)=20:15:12$$

4. Solution: 1.
Note the pattern of the remainders when these numbers is divided by 6:
1, 3, 3, 1, 3, 3, 1, 3, 3, …. Since 2011 ÷ 3 = 670 *r* 1, the remainder is 1.

5. Solution: 60 days.
$100\times\frac{3}{3+2}=60$ days.
3 days fishing every 5 days. 100 ÷ 5 × 3=60.

6. Solution: 50.
1800 ÷ 6 ÷ 6 = 50.

7. Solution: 879.
Method 1:
$\overline{abc}=100a+10b+c$.
$\overline{cba}=100c+10b+a$.
$100c+10b+a-100a+10b+c\ =\ 99(c-a)=99$.
So $c-a=1$. We know that *a*, *b*, and *c* are all distinct, so the greatest value of $\overline{abc}$ is 879.

Method 2:

$\overline{abc}+99=\overline{cba}$， $a=c+9$， To make $\overline{abc}$ the greatest, if $a=9$, then $c=0$， $\overline{cba}$ is not 3-digit number, contradiction. If $a=8$， then $c=9$， b =7， so $\overline{abc}$ =879。

8. Solution: 17.

Method 1:

(20 + 7 × 2) ÷ 2 = 17.

Method 2:

Let the number of apples in bags A and B be A and B, respectively.

$A + B = 20$ (1)

$A - 7 = B + 7$ (2)

$A + B = 20$

(1) + (2): $2A = 34$ $\Rightarrow$ $A = 17$.

9. Solution: 34.

Group is 18, 19, 25, 26. 1 + 8 + 1 + 9 + 2 + 5 + 2 + 6 = 34

						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	29	29
30	31					

10. Solution: $4.

The first way: (100 – 20) × 0.8 = $64

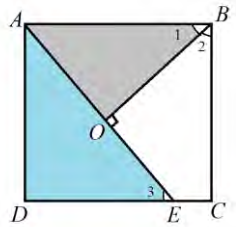
The second way: 100 × 0.8 – 20 = $60.

64 – 60 = $4.

11. Solution:

Method 1:

we label some angles as shown in the figure.

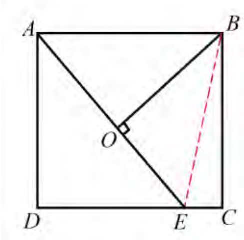


In $OECB$, $\angle 2 + \angle OEC = 180°$. Since $\angle 3 + \angle OEC = 180°$, $\angle 2 = \angle 3$, $\angle 1 = \angle DAE$. ΔABO is similar to ΔEAD.

Thus $\frac{AB}{AE} = \frac{OB}{AD} \Rightarrow \frac{12}{AE} = \frac{9}{12} \Rightarrow AE = 16$.

Method 2:

Connect BE.



$S_{\Delta ABE} = \frac{1}{2} S_{ABCD} = \frac{1}{2} \times 12 \times 12 = 72$

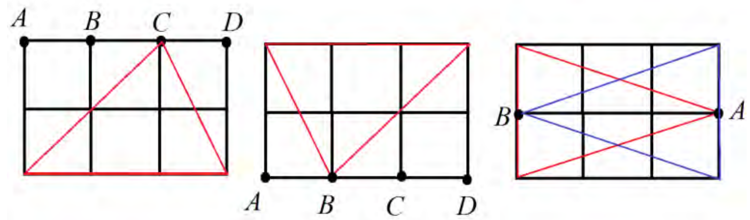
We see that OB is the height of ΔABE. Thus

$S_{\Delta ABE} = \frac{1}{2} AE \times OB = \frac{1}{2} AE \times 9$.

So $\frac{1}{2} AE \times 9 = 72 \Rightarrow AE = 16$.

12. Solution: 10.

$6 = 1 \times 6 = 2 \times 3$. when the base is 3, we have $4 \times 2 = 8$. When the base is 2, we have $1 \times 2 = 2$. So total $8 + 2 = 10$ different triangles with an area of 3 cm^2.



13. Solution: 9.

If one person is seated, at most we can have two empty seats going with that person. So at most one person can reduce three seats. 27/3 = 9. The least value of n is 9.

The following arrangement works. Each letter represents one person and each square represents one empty seat.

□A□□B□□C□□D□□E□□F□□G□□H□□I□

14. Solution: 30.

We divide the 9 numbers into three groups:

Group 1:	1,	4,	7	(the remainder is 1 when divided by 1)
Group 2:	2,	5,	8	(the remainder is 1 when divided by 2)

Group 3: 3, 6, 9 (the remainder is 1 when divided by 0)

Case 1: We take one number from each group. The sum is a multiple of 3. The number of ways to do so is $\binom{3}{1}\times\binom{3}{1}\times\binom{3}{1}=27$ ways.

Case 2: We take three numbers from each group. The sum is a multiple of 3. The number of ways to do so is $\binom{3}{3}+\binom{3}{3}+\binom{3}{3}=3$ ways.

The answer is 27 + 3 = 30 ways.

15. Solution: 15.

Using the worst case scenario, we take out 4 red balls first. Next we take out 5 yellow balls. Finally we take out 5 black balls. Now we only need to take out one balls to guarantee getting 6 balls of the same color. The answer is 4 + 5 + 5 + 1 = 15.

16. Solution: 5.

The number of candies Alex took must be a multiple of 3. (3 + 4 + 5 + 7 + 9 + 13) = 41.

41 ÷ 3 = 13 r 2.

So the number of candy in the bag left must have a remainder of 3 when divided by 3.

We see that only 5 has the remainder 2 when divided by 3 among the numbers 3, 4, 5, 7, 9, 13. So the number of candy in the bag left on the table is 5.

Method 2:

$13 + 7 + 4 = 2\times3 + 9$.

So the number of candy in the bag left on the table is 5.

17. Solution: 34.

Let a be Bob's age two years ago. $4a$ is Alex's age two years ago.

Two years from now Bob's age will be $a + 2 + 2 = a + 4$.

Two years from now Alex's age will be $4a + 2 + 2 = 4a + 4$.

$4a + 4 = 3(a + 4) \quad \Rightarrow \quad a = 8$

Two years from now Alex's age will be $4a + 2 + 2 = 4a + 4 = 34$.

So now Alex is 34 – 2 = 32 years old.

18. Solution: 1.
Method 1:
$(4 \times 30 - 110) \div (4 - 3) = 10$

Method 2
Let c and a be the numbers of toy cars and airplanes, respectively.

$c + a = 30$ (1)
$4c + 3a = 110$ (2)

$(1) \times 4 - (2)$: $a = 10$.

19. Solution: $\frac{3}{4}a - 5 + \frac{1}{2}b = \frac{1}{4}(3a + 2b) - 5 = 1$.

20. Solution:
Method 1:
($6.90 + $22.80) = $29.7 can buy (3 + 8) = 11 pounds of apples and 2 + 9 = 11 pounds of oranges. So the cost of 1 pound of apples and 1 pound of oranges is $29.7 ÷ 11 = $2.7.

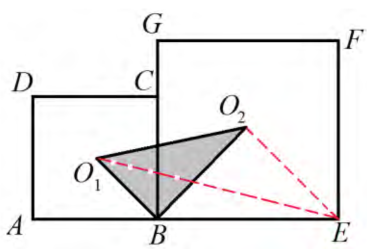
Method 2:
Let a and o be the costs of 1 pound of apple and 1 pound of orange, respectively.

$3a + 2o = 6.90$ (1)

$8a + 9o = 22.80$ (2)

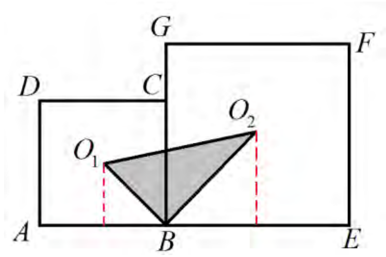
(1) + (2): $11(a + o) = 29.70 \Rightarrow a + o = 29.70 \div 11 = \$2.7$.

21. Solution:
Method 1:
Connect O_1E, O_2E. We get $\angle O_1BA = \angle O_2EA = 45°$.
So $O_1B \parallel O_2E$, and



$$S_{\Delta O_2O_1B}=S_{\Delta O_1BE}=\frac{1}{2}\times BE\times\frac{1}{2}\times AD=\frac{1}{2}\times 6\times\frac{1}{2}\times 4=6.$$

Method 2:
The difference between the areas of one right trapezoid minus two right triangles.
$(2+3)\times 5\div 2-2\times 2\div 2-3\times 3\div 2=6$.



22. Solution: 32°.
When the time is 4 pm, the angles formed by the minutes hand and the hour hand is 120°.
The minute hand moves 6° and the hour hand moves 0.5° in one minute.
After 16 minutes, the angle formed by two hands reduces $(6-0.5)\times 16=88°$.
The answer is $120-88=32°$.

23. Solution: 2008.

$$\frac{1}{2072}+\frac{1}{65009}=\frac{1}{8\times 259}+\frac{1}{251\times 259}=\frac{251+8}{8\times 251\times 259}=\frac{259}{8\times 251\times 259}=\frac{1}{8\times 251}=\frac{1}{2008}.$$

So $A=2008$.

24. Solution: 60 days.
Let B be the number of days for Bob to finish the job alone.
When Bob works for 10 days, he finishes the job: $\frac{1}{B}\times 10$

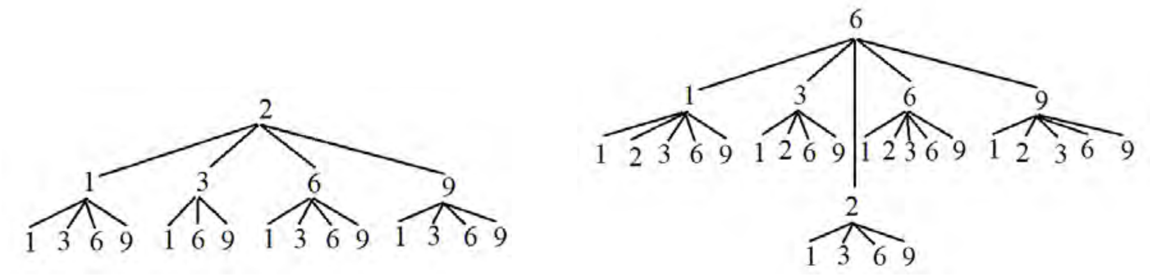
$$\left(\frac{1}{B}+\frac{1}{40}\right)\times 20=1-\frac{1}{B}\times 10 \quad\Rightarrow\quad \frac{1}{B}\times 30=\frac{1}{2} \quad\Rightarrow\quad B=60 \text{ days.}$$

25. Solution: 7.

$$\overline{abc}+\overline{cba}=100(a+c)+20b+(a+c)=101(a+c)+20b=888.$$

We see that the units digit of $20b$ is 0. So $a+c=8$ and $b=4$.
We know that both a and c are digits. Thus there are 7 solutions to $a+c=8$.
So we have 7 such 3-digit positive integers: 147, 246, 345, 444, 543, 642, and 741.

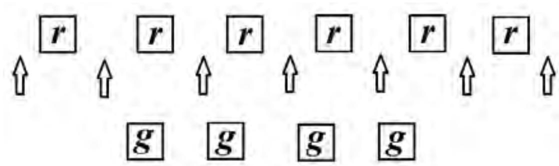
26. Solution: 38.
To make an even integer, the units digit must be even (2 or 6). When the units digit is 2, we have 15 such numbers. When the units digit is 6, we have 23 such numbers. 15 + 23 = 38.



27. Solution:
24300 × (1 – 10% – 24% – 12% – 36%) = 4374.

28. Solution: 60°.
$\angle BOD : \angle COD = 4:1$, so $\angle BOC : \angle COD = 3:1$. $\angle COD = 30°$. $\angle AOD = 60°$.

29. Solution: 35.
Alex can put 6 red balls in a row. there are seven spaces and then he can put four green balls between these red balls as shown in the figure. The number of ways is



$$\binom{7}{4} = \binom{7}{3} = \frac{7\times6\times5}{3\times2\times1} = 35.$$

30. Solution: 5.
The number of monkeys in the end should be the factor of 56 – 1 = 55: 1, 5, 11, or 55. So at first, the number of monkeys could be 5 – 4 = 1, 11 – 4 = 7, or 55 – 4 = 52.
Since at first a group of monkeys are dividing the pears, the number of monkeys should not be 1. 7 is a possible number fo the group of monkeys first dividing the pears. Then each monkey gets 55 ÷11 = 5 pears.

51 is not a possible number since 51 is not a factor of 56. So the answer is 5.

Solutions to Practice Test 12

1. Solution: 1300.
$13\times 23+1001 = 13\times 23+13\times 77=13(23+77)=13\times 100=1300$

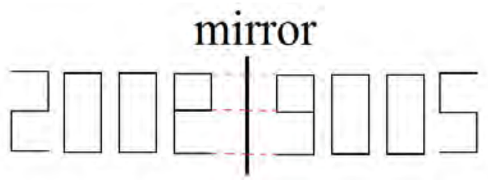
2. Solution: cm.
The sum of the perimeters of the two parts = the perimeters of the original rectangle + 2 × the length of the dashed line
= 2(12 + 10) + 2[10 + (12 - 3 - 4) × 3] = 44 + 50 = 94 cm.

3. Solution: 300.
(1 + 3 + 5 + … + 19) × 3 = $\dfrac{(1+19)\times 10}{2}\times 3=300$

4. Solution: 60.
When four teachers are all 30 years old, the fourth person gets the greatest possible age.
36 × 5 − 30 × 4 = 60.

5. Solution: 63.
$7a\times 9b=11\times 13 \quad\Rightarrow\quad 63ab=143 \quad\Rightarrow\quad 143\div ab=63$.

6. Solution: 9005.
Method 1:
These numbers are symmetrical about the line (mirror).



Method 2:
Turn the page over and read the number.

7. Solution: 10.
120 = 17 × 7 + 1 = 17 × 6 + 18 = 17 × 5 + 35 = 17 × 4 + 53 = 17 × 4 + 11 × 4 + 4 × 2.
The answer is 4 + 4 + 2 = 10.

8. Solution: 3.
The units digit can only be 1, 3, 5, 7, or 9 for any odd numbers. If one of the three consecutive odd positive integers is 5, the product of them will be 5. If the units digits of the three consecutive odd positive integers are 7, 9, and 1, respectively, the units digit of the product of them will be 3. If the units digits of the three consecutive odd positive integers are 9, 1, and 3, respectively, the units digit of the product of them will be 7. So the smallest units digit of the product of three consecutive odd positive integers is 3.

9. Solution: 81.
Since the 2-digit positive integer has an odd number of divisors, it must be a square number. We have two numbers: 16 and 81. The answer is 81,

10. Solution: 133.
If an integer are multiples of 2 , it is a multiple of $2 \times 3 = 6$.
$1000 \div 6 = 166\ r\ 4$. So there are 166 numbers from 1 to 1000 that are multiples of 2 and 3.
Since 5 and 6 are relatively prime, a number is a multiple of 5 if it is a multiple of $5 \times 6 = 30$.
$1000 \div 30 = 33\ r\ 10$.

So there are $166 – 33 = 133$ numbers from 1 to 1000 are multiples of 2 and 3 but not 5.

11. Solution: 73.
The total score of the 23 students is $23 \times 72 = 1656$.
The new average score for the class is $\frac{1656+96}{23+1}=\frac{1752}{24}=73$.

12. Solution: 22.
If a number is divisible by 6, it is divisible by both 2 and 3. So all the 3-digit numbers must be consist of 2, 4, 6, and 8. The sum of the digits of each number is divisible by 3.
So we have
246 (6 numbers by permutation).
228, 288 (6 numbers by permutation).
468 (6 numbers by permutation).
222

444
666
888
The answer is 6 + 6 + 6 + 1 + 1 + 1 + 1 = 22.

13. Solution:
$m = 3n + 24$ (1)
$m = 5n + 14$ (2)
(1) – (2): $2n = 10 \quad \Rightarrow \quad n = 5$.
Substituting the value of n into (1): $m = 39$.

14. Solution: 19.
Let Bob's age be x. Alex's age is $7x$. Three years ago Bob's age is $x – 3$ and Alex's age is $7x – 3$. So $10(x – 3) = 7x – 3 \quad \Rightarrow \quad x = 9 \quad \Rightarrow \quad 7x = 63$.
Three years ago Bob's age is $x – 6 = 3$ and Alex's age is $7x – 6 = 57$.
$57 \div 3 = 19$.

15. Solution:
Method 1:
After easting Peter and Sam together have 24 - 3 – 2 = 19 candies. Sam will have (19 – 3) ÷ 2 = 8 candies. Originally Sam has 8 + 2 = 10 candies and Peter has 24 – 10 = 14 candies.

Method 2:
Let P be the number of candies Peter has and S be the number of candies Sam has originally.
$P + S = 24$ (1)
$P – 3 = S – 2 +3$ (2)

(1) + (2): $P = 14$.

16. Solution: 2 meters.
The perimeter of the tree square is $4 \times 104 = 416$.
The distance between any two trees is $416 \div 218 = 2$ meters.

17. Solution: 3520.
Let x be the number of parts 11 machines can produce in 8 hours.

$$\frac{600}{5\times 3}=\frac{x}{11\times 8} \quad \Rightarrow \quad x=3520.$$

18. Solution: 30 minutes.
Let A be Alex's speed and B be Bob's speed. Let t be the time needed for Bob to catch Alex.

$(B - A)\ t = 30 \times A \quad \Rightarrow \quad (2A - A)\ t = 30 \times A \quad \Rightarrow \quad t = 30$ minutes.

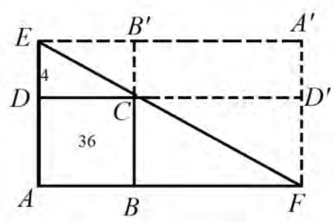
19. Solution: 66.
The total amount is 10 × 2 + 10 × 5 = 70.
Since there is no 1 cent coin, there are no 1 cent and 3 cents values possible.
Also there are no values for 70 – 1 = 69 cents and 70 – 3 = 67 cents. We checked and all other values can be made. So the answer is 70 – 4 = 66.

20. Solution: 11: 25.
The starting time for each period is 8: 00, 8: 55, 9: 50, and 10: 45, respectively. The time is then 11: 25 when 4th period is finished

21. Solution: 9.
Method 1:
Since $ABCD$ is a square with the area of 36, its side length is 6.
Let BF be x.
We know that $S_{\Delta CDE}+S_{ABCD}+S_{\Delta BCF}=S_{\Delta AEF}$. Thus

$$\frac{1}{2}\times 4\times 6+36+\frac{1}{2}\times 6x=\frac{1}{2}\times(4+6)\times(6+x) \Rightarrow \quad 40+3x=30+5x \quad \Rightarrow \quad x=9.$$

Method 2:
We create a rectangle as shown in the figure.



We know that $S_{\Delta CDE}=S_{\Delta B'CE}$. $S_{\Delta CBE}=S_{\Delta CD'F}$. $S_{\Delta AFE}=S_{\Delta A'FE}$.

So $S_{ABCD}=S_{B'CD'A'}$.

Thus $BCD' = BF = 36 \div 4 = 9$.

22. Solution: 360°.

$$\angle 1 + \angle 2 + \angle 3 = (180° - \angle ABC) + (180° - \angle BCA) + (180° - \angle CAB)$$
$$= 3 \times 180° - (\angle ABC + \angle BCA + \angle CAB) = 3 \times 180° - 180° = 360°$$

23. Solution: 91.

2011 = 99 × 20 + 31 = 98 × 20 + 51 = 97 × 20 + 71 =96 × 20 + 91.

24. Solution: 76.

Total we have 5 + 3 + 2 = 10 fruits. We have $\binom{10}{2} = 45$ ways to take out two fruits.

We have $\binom{5}{1} \times \binom{3}{1} + \binom{3}{1} \times \binom{2}{1} + \binom{2}{1} \times \binom{5}{1} = 15 + 6 + 10 = 31$ ways to take out two different fruits.

The answer is 45 + 31 = 76.

25. Solution: 437.

We know that $80008 \le \overline{8abc8} \le 89998$.

80008 ÷ 2009 = 39 *r* 1657

89998 ÷ 2009 = 49 *r* 1602

So $2009 \times 40 \le \overline{8abc8} \le 2009 \times 44$

We notice that the last digit of is 8. The last digits are 0, 9, 8, 7, and 6 of the products of 2009 with 40, 41, 42, 43, and 44, respectively.

So we have $\overline{8abc8} = 2009 \times 42 = 84378$. Therefore $\overline{abc}$ = 437.

26. Solution: 428571.

Let x be the 5-digit number when the digit 1 is removed. The original 6-digit number is $(10x + 1)$. The new 6-digit number is $(100000 + x)$.

$3(100000 + x) = 10x + 1 \quad \Rightarrow \quad x = 42857$.

So the original number is 428571.

27. Solution: 10.

Considering the worst case. We take out 1 purple ball, 2 green balls, contains 3 red balls , and 3 yellow balls.

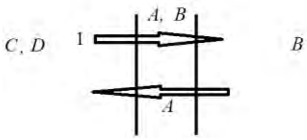
Now no matter what color the next ball is, we satisfy the condition. So the answer is 1 + 2 + 3 + 3 + 1 = 10.

28. Solution: 13 minutes.

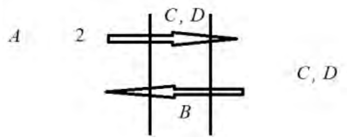
We need to use cow A as many times as we can. We need to use cow D as few times as we can.

One configuration is as follows:

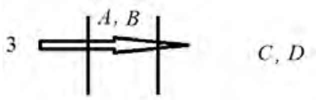
Step 1: 2 + 1 = 3 minutes



Step 2: 6 + 2 = 8 minutes



Step 3: 2 minutes



Total time is 3 + 8 + 2 = 13 minutes.

29. Solution: 180.

We have two cases:

Case I:

Bob gets the puzzle book. $\binom{1}{1}=1$.

Catherine gets the camera. $\binom{1}{1}=1$

We have 5 gifts left and we have $\binom{5}{3}=10$ ways to select 3 gifts and we have 3! = 6 ways to order them. So we have 1 × 1 × 10 × 6 = 60 ways.

Case II:

Bob gets the toy car. $\binom{1}{1}=1$

Catherine gets the either puzzle book or camera. $\binom{2}{1}=2$

We have 5 gifts left and we have $\binom{5}{3}=10$ ways to select 3 gifts and we have 3! = 6 ways to order them. So we have 1 × 2 × 10 × 6 = 120 ways.

The answer is 60 + 120 = 180.

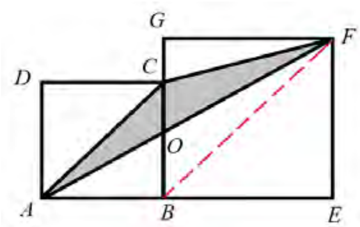
30. Solution:

Connect BF.

Method 1:

$$S_{\Delta ABF}=\frac{1}{2}AB\cdot EF$$

$$S_{\Delta BCF}=\frac{1}{2}BC\cdot GF$$



Since $AB = BC$ and $EF = GF$, $S_{\Delta ABF} = S_{\Delta BCF}$.

Since $S_{\Delta BOF} = S_{\Delta BOF}$, $S_{\Delta AOB} = S_{\Delta OCF}$.

So the shaded area is the same as $S_{\Delta ABC}=10$.

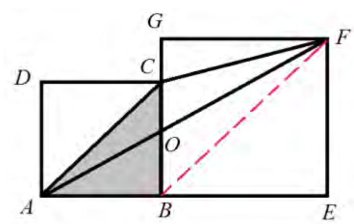
Thus the area of $ABCD$ is 2 × 10 = 20.

Method 2:

We see that $AC//BF$. So $S_{\Delta AOB} = S_{\Delta OCF}$.

So the shaded area is the same as $S_{\Delta ABC}=10$.

Thus the area of $ABCD$ is 2 × 10 = 20.



Solutions to Practice Test 13

1. Solution: $\frac{2010}{2011}$.

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{2010\times 2011} = \frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\ldots+\frac{1}{2010}-\frac{1}{2011}$$
$$= 1-\frac{1}{2011}=\frac{2010}{2011}$$

2. Solution: 18063.

Method 1:

$$(1\underbrace{00\cdots0}_{2007s\ \ 0}-1)\times\underbrace{55\cdots5}_{2007s\ \ 5}=\underbrace{55\cdots5}_{2007s\ \ 5}\underbrace{00\cdots0}_{2007s\ \ 0}-\underbrace{55\cdots5}_{2007s\ \ 5}=\underbrace{55\cdots5}_{2006s\ \ 5}\underbrace{44\cdots4}_{2007s\ \ 4}5.$$

So the sum of the digits is 5 × 2006 + 4 × 2007 + 5 = 9 × 2007 =18063.

Method 2:

We see that 9 × 5 = 45. The sum of the digits of 45 is 4 + 5 = 9.

99 × 55 = 5445. The sum of the digits of 5445 is 5 × 2 + 4 × 2 = 9 × 2 =18.

999 × 555 = 55445. The sum of the digits of 5445 is 5 × 3 + 4 × 3 = 9 × 3 =27.

So the sum of the digits in the original product is 9 × 2007 = 18063.

3. Solution: 42/77.

Method 1:

When 2 is added to the numerator of a fraction, the result is $\frac{4}{7}$. So the original fraction

can be $\frac{6}{14}$, $\frac{10}{21}$, $\frac{14}{28}$, $\frac{18}{35}$, $\frac{22}{42}$, $\frac{26}{49}$, $\frac{30}{56}$, $\frac{38}{70}$, $\frac{42}{77}$, $\frac{46}{84}$, …,

When 2 is subtracted from the denominator of the fraction, the result is $\frac{14}{25}$. So the

original fraction can be $\frac{52}{28}$, $\frac{42}{77}$, $\frac{56}{102}$, ….

We see that $\frac{42}{77}$ satisfies both conditions. So the answer is $\frac{42}{77}$.

Method 2:

Let the original fraction be $\frac{ma}{mb}=\frac{a}{b}$.

So $\frac{a+2}{b}=\frac{4}{7} \quad \Rightarrow \quad 7a+14=4b \quad \Rightarrow \quad 14a+28=8b$ (1)

So $\frac{a}{b-2}=\frac{14}{25} \quad \Rightarrow \quad 25a=14b-28 \quad \Rightarrow \quad 25a+28=14b$ (2)

(2) − (1): We see that $11a=6b$ satisfies both conditions. So the answer is $\frac{a}{b}=\frac{6}{11}$.

Since 11 and 7 are relatively prime, we know that $\frac{ma}{mb}=\frac{7\times 6}{7\times 11}=\frac{42}{77}$.

4. Solution: 2011.

We know that even + even = even. So a^2 must be even.

We know that even × even = even. So a must be an even prime. So a = 2.

Thus $2^2 + b = 2011 \quad \Rightarrow \quad b = 2007$.

$a + b + 2 = 2 + 2007 + 2 = 2011$.

5. Solution: 7.

$\left\lfloor \frac{30}{5^1} \right\rfloor + \left\lfloor \frac{30}{5^2} \right\rfloor = 6+1=7$.

6. Solution: 59.

Let the three numbers be x, y, and z.

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1155}{2006} \quad \Rightarrow \quad \frac{yz+zx+xy}{xyz}=\frac{1155}{2006}$.

We see that 2006 = 2 × 17 × 59 and 2 × 17 + 2 × 59 + 17 × 59 = 1155.

So the greatest number of the three primes is 59.

7. Solution: 45.

Let the original number be $\overline{ab}=10a+b$. The new number is $\overline{a0b}=100a+b$.

So $100a + b = 9(10a + b) \quad \Rightarrow \quad 10a = 8b \quad \Rightarrow \quad 5a = 4b$.
We know that both a and b are digits. So $a = 4$ and $b = 5$. The original number is 45.

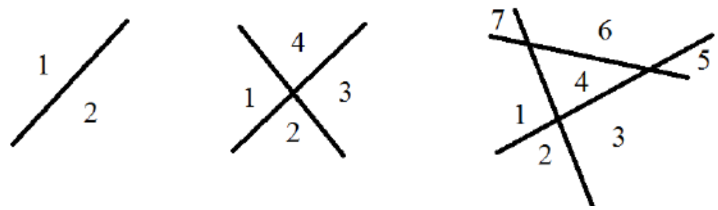
8. Solution: $\frac{14}{26}$.

$\frac{7}{13} = \frac{14}{26} = \frac{21}{39} = \cdots$

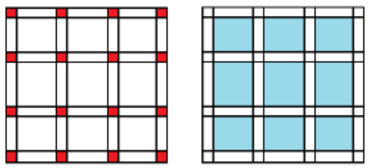
26 – 14 = 12; 39 – 21 = 18 > 12.

So the fraction is $\frac{14}{26}$.

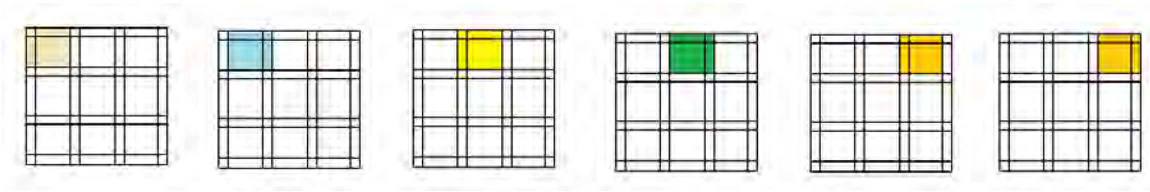
9. Solution: 56.
1 line can divide the plan into 1 + **1** parts;
2 lines can divide the plan into 1 + 1 + **2** parts;
3 line can divide the plan into 1 + 1 + 2 + **3** parts;
…
10 line can divide the plan into 1 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + **10** = 56 parts;



10. Solution:
We count by 1:
16 + 9 = 25 squares.

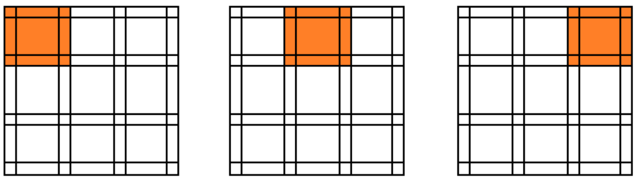


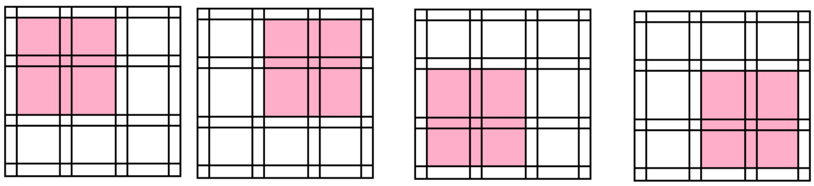
We count by 4:



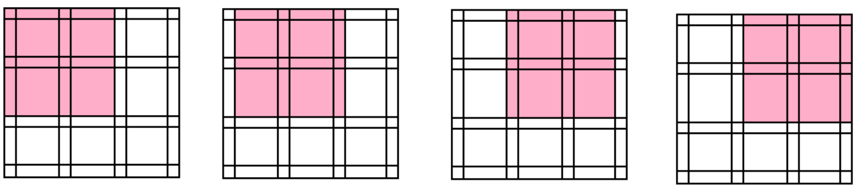
$6 \times 6 = 36$ squares.

We count by 9:
$3 \times 3 + 1 \times 4 = 13$ squares.

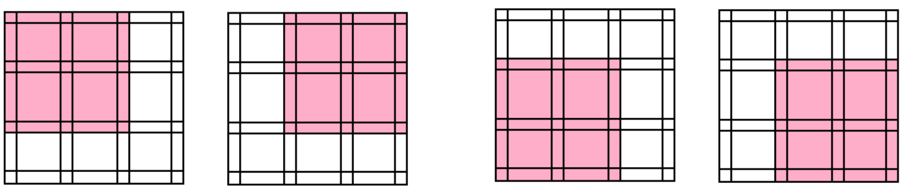


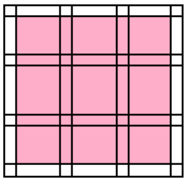


We count by 16:
$4 \times 4 = 16$ squares.

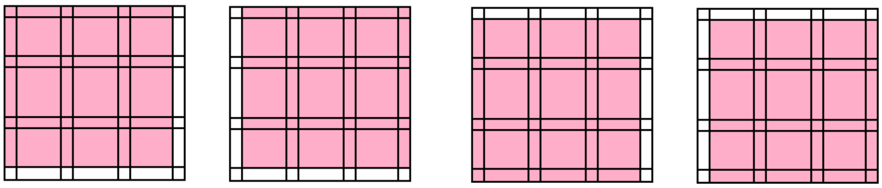


We count by 25:
$2 \times 2 + 1 = 5$ squares.

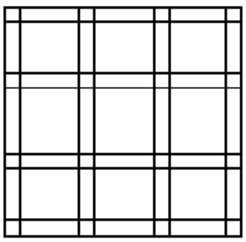




We count by 36:
$2 \times 2 = 4$ squares.



We count by 49:
1 square.



$25 + 36 + 13 + 16 + 5 + 4 + 1 = 100$.

11. Solution: 10968.

Let N be $\overline{abcde}$. Another 5-digit number be $\overline{e'd'c'b'a'}$. All the digits are among 0, 1, 6, 8, and 9.

So $e' - a = 7$. We know that $9 - 2 = 7$ and $8 - 1 = 7$. We see that $e' = 9$ and $a = 7$ will not work because the digit 7 is not usable. Thus that $e' = e = 8$ and $a = a' = 1$.

Consider the hundred digit, $c' - c = 6$. So $c' = 9$ and $c = 6$.

Since $d' - 1 - b = 8$, $d' - b = 9$. We conclude that $d' = 9$ and $b = 0$.

Thus we know that the original number is 10968.

12. Solution: 671.

2011 ÷ 3 = 670 *r* 1.

There are 670 + 1 = 671 kids said "1" starting from left to right. There are 670 + 1 = 671 from right to left also saying 1. These kids saying "1" twice are the same kids.

13. Solution: 39.

Let *A*, *B*, and *C* be the numbers of pieces of candy in boxes *A*, *B*, and *C*, respectively.

$A + B + C = 55$ (1)

$A = 3B \qquad \Rightarrow \qquad B = \frac{1}{3}A$ (2)

Substituting (2) into (1): $A + \frac{1}{3}A + C = 55 \qquad \Rightarrow \qquad 4A + 3C = 165$ (3)

The smallest value of *C* is 3. thus *A* = 39.

14. Solution: 19.

$361 = 19 \times 19$. So at least 19 boats are needed.

15. Solution: 0.6 kg.

$0.2 \div (1 - \frac{1}{3} - \frac{1}{3}) = 0.6$.

16. Solution: 3705.

$$99 \div (\frac{1}{5\frac{5}{12}} - \frac{1}{6\frac{1}{3}}) = 99 \div (\frac{12}{65} - \frac{3}{19}) = 99 \times \frac{65 \times 19}{12 \times 19 - 65 \times 3} = 3705.$$

17. Solution: 9.

Method 1:

Let t be the number of hours needed for Charles joined Bob to finish the job together.

$$(\frac{1}{10} + \frac{1}{15}) \times 2 + \frac{1}{15} \times 3 + (\frac{1}{15} + \frac{1}{20}) \times t = 1 \qquad \Rightarrow \qquad t = 4.$$

The answer is 2 + 3 + 4 = 9.

Method 2:

$$[1-(\frac{1}{10}+\frac{1}{15})\times 2-\frac{1}{15}\times 3]\div(\frac{1}{15}+\frac{1}{20})+(2+3)=\frac{7}{15}\div\frac{7}{60}+5=9$$

18. Solution: 42.

$[15+(5-2)]\div(5-3)=9$.

$3\times 9+15=42$.

19. Solution: $40,000.

Let A be Alex's money and B be Bob's money originally.

$A + B = 150,000$ (1)

$1.2\,A = 3(0.8B)$ $\Rightarrow$ $A = 2B$ (2)

Substituting (2) into (1): $2B + B = 150,000$ $\Rightarrow$ $B = 50,000$.

The answer is $0.8B = 0.8 \times 50,000 =$ $40.000.

20. Solution: 150 grams.

We introduce the "*C-V-S*" method. *C* is the concentration or the strength of the solution.

Name	C	×	V	=	S
A	0.2	×	x	=	$0.2x$
			+		+
B	0.05	×	y	=	$0.05y$
			||		||
Mixture	0.15	×	450	=	0.15×450

$x + y = 450$ $\Rightarrow$ $0.2x + 0.2y = 450 \times 0.2$ (1)

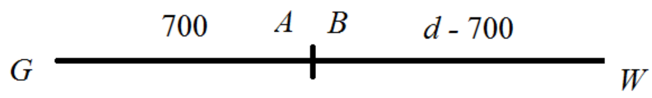
$0.2x + 0.05y = 0.15 \times 450$ (2)

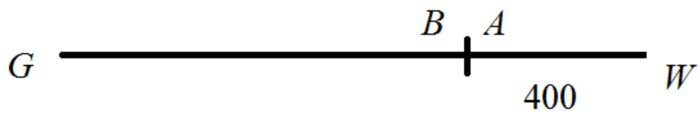
(1) – (2): $0.15y = 22.5$ $\Rightarrow$ $y = 150$ grams.

21. Solution: 1700 meters.

When they first meet, Alex travels 700 meters. The sum of the meters travelled by Alex and Bob is *d*, the distance from Greenville to Winterville. When they meet the second time, the sum of the meters travelled by Alex and Bob is 3*d*.

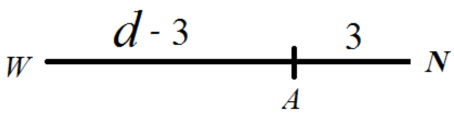
When they travel *d* meters, Alex travels 700 meters. When they travel 3*d* meters, Alex travels 3 × 700 = 2,100 meters.





So $d - 700 + 400 = 1400 \quad \Rightarrow \quad d = 1700$ meters.

22. Solution: 15 km.
Let V_A be the speed of the ship against the current and V_C be the speed of the ship with the current, d be the distance from Washington to New Bern.
In first hour, the ship travels from Washington to point A, which is 3 km from new Bern.



$$\frac{d-3}{V_A} = 1 \quad \Rightarrow \quad V_A = d - 3 \qquad (1)$$

$$V_C = V_A + 8 \quad \Rightarrow \quad V_C = d - 3 + 8 = d + 5 \qquad (2)$$

$$\frac{3}{V_A} + \frac{d}{V_C} = 1 \qquad (2)$$

Substituting (1) and (2) into (3): $\frac{3}{d-3} + \frac{d}{d+5} = 1 \quad \Rightarrow \quad \frac{3}{d-3} + \frac{d}{d+5} = 1 \Rightarrow$

$$\frac{3}{d-3} = 1 - \frac{d}{d+5} \quad \Rightarrow \quad \frac{3}{d-3} = \frac{5}{d+5} \quad \Rightarrow \quad \frac{3}{d-3} = \frac{5}{d+5} = \frac{2}{8} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{3}{d-3} = \frac{1}{4} \quad \Rightarrow \quad d - 3 = 12 \quad \Rightarrow \quad d = 15\,\text{km}.$$

23. Solution: 12.
Method 1:
Let A be the age for Alex now and B be the age for Bob now. Let x be the number of

years later that Bob's age will be $\frac{5}{12}$ of Alex's age.

$$\frac{5}{12}(A+x)=B+x \quad \Rightarrow \quad \frac{5}{12}(36+x)=8+x \quad \Rightarrow \quad 15+\frac{5x}{12}=8+x \quad \Rightarrow$$
$$\frac{7x}{12}=7 \quad \Rightarrow \quad x=12.$$

So 12 years later Bob's age will be $\frac{5}{12}$ of Alex's age.

Method 2:

(36 – 8) ÷ (12 – 5) × 5 – 8 = 28 ÷ 7 × 5 – 8 = 12.

So 12 years later Bob's age will be $\frac{5}{12}$ of Alex's age.

24. Solution: 5 hours.

Method 1:

Let x be the amount of water in the pond, y be the rate of water coming into the pond, and m be the number of hours needed 25 pumps to empty the pond.

$$\frac{x+20y}{10\times 20}=\frac{x+10y}{15\times 10}=\frac{x+my}{25\times m} \qquad (1)$$

From (1), we get

$$\frac{x+20y}{10\times 20}=\frac{x+10y}{15\times 10}=\frac{10y}{50}=\frac{y}{5}=\frac{10y-my}{50-25m} \quad \Rightarrow \quad \frac{y}{5}=\frac{10y-my}{50-25m} \quad \Rightarrow \quad 1=\frac{10-m}{10-5m}$$
$$\Rightarrow \quad 10-5m=10-m \quad \Rightarrow \quad 4m=20 \quad \Rightarrow \quad m = 5 \text{ hours.}$$

Method 2:

(10 × 20 – 15 × 10) ÷ (20 – 10) = 50 ÷ 10 = 5 pumps.

(10 – 5) × 20 ÷ (25 – 5) = 100 ÷ 20 = 5 hours.

25. Solution: 160 seconds.

The toy car travels 2 + 3 meters every 2 seconds.

400 ÷ 5 = 80.

2 × 80 = 160 seconds.

26. Solution: $30.

$A + B + C + D = 190$ (1)
$A + 10 = B - 20 = 2C = 0.5D$ (2)

From (2), we have
$B = A + 30$
$C = 0.5(A + 10)$
$D = 2(A + 10)$
Substituting these values into (1): $A + A + 30 + 0.5(A + 10) + 2(A + 10) = 190$
$\Rightarrow$ $4.5A + 55 = 190$ $\Rightarrow$ $4.5A = 135$ $\Rightarrow$ $A = \$30$.

27. Solution: $716.4.
Total number of bottles needed is $1194 \times 2 = 2388$.
We know that 6 empty bottles can be used to exchange for one drink. $6 \times 5 = 30$ and $30 + 1 - 1 = 30$.
So for every 30 drinks, the principle needs at least to pay $8 \times (30 - 5) = \$45$.
Total saving is $1.8 \times 2388 - 45 \times \dfrac{2388}{30} = \$716.4$.

28. Solution: cm^2.
$S_{\Delta BDC} - S_{\Delta ABD} = 10 \quad \Rightarrow \quad \dfrac{BC \times h}{2} - \dfrac{AD \times h}{2} = 10$
$\Rightarrow \quad h(BC - AD) = 20 \quad \Rightarrow \quad 5h = 20 \quad \Rightarrow \quad h = 4$.
$S_{ABCD} = \dfrac{(AD + BC) \times h}{2} = \dfrac{15 \times 4}{2} = 30$ cm^2.

29. Solution: 91.
Starting from the second layer, there are 6 more small hexagons with each increased layer.
The answer is $1 + 6 + 6 \times 2 + 6 \times 3 + 6 \times 4 + 6 \times 5 = 91$ bees.

30. Solution: 96 ways.
We color A first and we have 4 ways to color. Then we color B, D, and E with 3, 2, 2 ways, respectively. We color C last with 2 ways. The answer is $4 \times 3 \times 2 \times 2 \times 2 = 96$ ways.

Solutions to Practice Test 14

1. Solution: 80.

$$3.6 \times (2\frac{1}{2011} + 7\frac{2010}{2011}) \times \frac{4}{3} \times \frac{5}{3} = 36 \times \frac{4}{3} \times \frac{5}{3} = 80.$$

2. Solution: 20.

$$20.05 = 20 + 0.05 = 20 + \frac{5}{100} = 20 + \frac{1}{20}.$$

So the positive integer is 20.

3. Solution: (35, 8100).

38 = 1 + 1 + 1 + 35 = 10 + 10 + 9 + 9.

The smallest product is 1 × 1 × 1 × 35 = 35.

The greatest product is 9 × 9 × 10 × 10 = 8100.

4. Solution:

a – 1 = 2011 – 1 = 2010 = 2 × 1005

a = 2011

a + 1 = 2011 + 1 = 2012 = 2 × 1006

a + 2 = 2012 + 1 = 2013 = 3 × 671.

2011 is not divisible by any prime number less than 45.

The answer is 3.

5. Solution: $\frac{71}{105}$.

The only value of p such that p, $p + 2$, and $p + 4$ are prime numbers is 3.

So $\frac{1}{p} + \frac{1}{p+2} + \frac{1}{p+4} = \frac{1}{3} + \frac{1}{5} + \frac{1}{7} = \frac{35+21+15}{105} = \frac{71}{105}$.

6. Solution: 24.

2 + 4 + 6 + 8 = 20.

The number of quadruples (a, b, c, d) is 4! = 24.

7. Solution: 2011.

Method 1:

Let sum of the middle two positive integers $x + y$.

$$\frac{x+y}{2}=\frac{10055}{10} \quad \Rightarrow \quad x + y = 2011.$$

Method 2:

Let the smallest number be x.

$$x+x+1+x+2+x+3+x+4+x+5+x+6+x+7+x+8+x+9=10055$$

$$\Rightarrow \quad 10x + 45 = 10055 \Rightarrow \quad x = 1001 \quad \Rightarrow x+4+x+5=1001\times 2+9=2011.$$

8. Solution: 60/7.

Method 1:

LCM(15, 10, 20) = 60.

GCF(14, 21, 49) = 7.

The answer is 60/7.

Method 2:

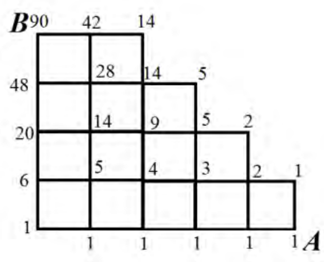
This question is the same as to find the least common multiple of $1\frac{1}{14}$, $\frac{10}{21}$, and $\frac{20}{49}$.

By the formula , we have:

$$LCM(\frac{15}{14},\frac{10}{21})=\frac{15\times 10}{GCF(15\times 21,14\times 20)}=\frac{150}{35}=\frac{30}{7}$$

$$LCM(\frac{30}{7},\frac{20}{49})=\frac{30\times 20}{GCF(30\times 49,7\times 20)}=\frac{600}{70}=\frac{60}{7}.$$

9. Solution: 90.

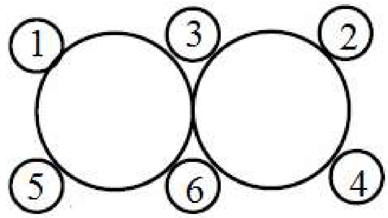


10. Solution:

$15 \times 2 - (1 + 2 + 3 + 4 + 5 + 6) = 30 - 21 = 9$.

The sum of the numbers inside the two small circles of the middle position is 9. The sum of the numbers inside the two small circle of the left end is $15 - 9 = 6$. The sum of the numbers in the two small circles of the right end is also 6.

One example is shown below.



11. Solution: 42.

Method 1:

Let x be the number of balls in the bag after the addition.

$$\frac{5}{12}(x-6) = \frac{1}{2}x - 6 \quad \Rightarrow \quad x = 42.$$

Method 2:

Let x be the number of balls in the bag before the addition.

$$\frac{\frac{5}{12}x + 6}{x+6} = \frac{1}{2} \quad \Rightarrow \quad x = 36.$$

$x + 6 = 42$.

Method 3:

Let the number of red marbles in the bag originally be 5 parts, and the number of other colored marbles in the bag originally be 7 parts. When 6 red marbles are added, the number of red marbles in the bag is 7 parts, and the number of other colored marbles in the bag is also 7 parts.

So the number of marbles in the bag after the addition is $6 \div 2 \times (7 + 7) = 42$.

12. Solution: 175.

Let x be the number of flowers blooming last year. The number of flowers not blooming last year was $2x + 55$.

So $x + 100$ is the number of flowers blooming this year. The number of flowers not blooming this year is $2x + 55 - 100$.

$x + 100 = 4(2x + 55 - 100)$ $\Rightarrow$ $x = 40$.

The total number of lowers is 40 + (2 × 40 + 55) = 175.

13. Solution: 50 kg.

$A + B = 110$ (1)

$\frac{1}{5}A + \frac{1}{4}B = 25$ (2)

(2) × 4: $\frac{4}{5}A + B = 100$ (3)

(1) – (3): $A = 50$.

14. Solution: 0.

(32 - 32) × 5 ÷ 9 = 0° C.

15. Solution: 21:1.

Method 1:

The first time he gives away $\frac{1}{21}$.

The second time he gives away $(1 - \frac{1}{21}) \times \frac{1}{20} = \frac{1}{21}$.

The third time he gives away $(1 - \frac{1}{21} - \frac{1}{21}) \times \frac{1}{19} = \frac{19}{21} \times \frac{1}{19} = \frac{1}{21}$.

So we see the pattern. The twentieth time he gives away $\frac{1}{21}$ of the original pile of oranges. The ratio is 21:1.

16. Solution: \$10.

Alex paid for shirt 1: 120 ÷ (1 + 0.2) = \$100.

Alex paid for shirt 1: 120 ÷ (1 – 0.2) = \$150.

The positive difference is (100 + 150) – 120 × 2 = \$10.

17. Solution: 2.4 hours.

The total number of hours is 4. Each court has two kids. Each person plays 4 × 3 × 2 ÷ 10 = 2.4 hours.

18. Solution: 60 kg.

Let Alex's money be "1 ".

$$\frac{1}{\frac{1}{40}+\frac{1}{60}+\frac{1}{120}}=20 \text{ kg.}$$

$20 \times 3 = 60$ kg.

19. Solution: 200 g.

Method 1:

Let C be the concentration or the strength of the solution, V be the volume of the solution. S is the substance of the solution.

Name	C	×	V	=	S
A	0.4	×	m	=	$0.4m$
			+		+
B	0.1	×	n	=	$0.1n$
			‖		‖
D	0.3	×	$m + n$	=	$0.3(m + n)$

$0.3(m + n) = 0.4m + 0.1n \quad\Rightarrow\quad 2n = m \quad (1)$

Name	C	×	V	=	S
D	0.3	×	$m + n$	=	$0.3(m + n)$
			+		+
E	0.2	×	300	=	60
			‖		‖
F	0.25	×	$300 + m + n$	=	$0.25(300 + m + n)$

$0.3(m + n) + 60 = 0.25(300 + m + n) \quad\Rightarrow\quad m + n = 300 \quad (2)$

Solving the system of equations (1) and (2): $m = 200$.

Method 2:

Let x be the weight in grams of the 30% salt solution.

$x \times 30\% + 300 \times 20\% = (300 + x) \times 25\% \quad\Rightarrow\quad x = 300$ grams.

Let y be the weight in grams of the 40% salt solution.

$y \times 40\% + (300 - y) \times 10\% = 300 \times 25\% \quad \Rightarrow \quad y = 200$ grams.

20. Solution: 1:15 pm.

$21 \div 6 = 3.5$ hours.

Alex rests 3 times (45 minutes) in 3.5 hours walking.

Total time on the road is 4 hours 15 minutes. The time is 1:15 pm (9:00 am + 4 hours 15 minutes).

21. Solution: : 5.25 km/h.

Method 1

The sum of their speeds is $4.5 \div 0.5 = 9$ km/h.

The difference of their speeds is $4.5 \div 3 = 1.5$ km/h.

Alex's speed is $(9 + 1.5) \div 2 = 5.25$ km/h.

(Bob's speed is $(9 - 1.5) \div 2 = 3.75$ km/h).

Method 2:

Let x be Alex's speed and y be Bob's speed.

$3(x - y) = 4.5 \qquad (1)$

$0.5(x + y) = 4.5 \qquad (2)$

Solving for x: $x = 5.25$ km/h.

22. Solution: 75.

Let Sara's speed be n times the speed of the escalator, and Peter's speed be $2n$ times the speed of the escalator.

When Peter walks down, his speed is $(2n - 1)$ times the speed of the escalator.

When Sara walks up, her speed is $(n + 1)$ times the speed of the escalator.

We know that Peter's speed is two times of Sara's speed. We also know that the number of stairs walked by Peter is two times of the number of steps walked by Sara. So the time of their walking is the same: $2n - 1 = n + 1 \quad \Rightarrow \quad n = 2$.

The number of stairs visible on the escalator when it is switched off is $50 + 50 \div 2 = 75$.

23. Solution:

Method 1:

By the time the cows have eaten all the grass, the total amount of grass consumed will equal to the amount of grass initially on the grassland plus the amount of grass re-growth with the time.

x = initial amount of grass in the grassland in one acre.
y = amount of grass growing in the grassland per acre per day.
z = the rate of each cow eating grass (let z be 1 part)
c = cows needed in the question

Suppose no cow was sold. These 4 cows will eat 4 × 2 (parts) = 8 parts of the grass.

$$\frac{x+30y}{17\times 30}=\frac{x+24y}{19\times 24}=\frac{x+4\times 2+8y}{c\times 8}=1$$

$$\frac{x+30y}{17\times 30}=\frac{x+24y}{19\times 24}=\frac{6y}{54}=\frac{y}{9}=1 \quad\Rightarrow\quad y = 9 \text{ parts.}$$

$$\frac{x+30y}{17\times 30}=1 \quad\Rightarrow\quad \frac{x+30\times 9}{17\times 30}=1 \quad\Rightarrow\quad x=17\times 30-9\times 30=8\times 30=240 \text{ parts.}$$

$$\frac{x+8+8y}{c\times 8}=1 \quad\Rightarrow\quad \frac{240+8+8\times 9}{c\times 8}=1 \quad\Rightarrow$$

$$8c=240+8+72=320 \quad\Rightarrow\quad c = 40.$$

Method 2:

Let the amount of grass consumed by one cow in one day be one part.
Since 17 cows can eat all of grass in 30 days the total amount of grass is

17 × 30 = 510 parts (1)

Since 19 cows can eat all of grass in 24 days the total amount of grass is
19 × 24 = 465 parts (2)

The difference between (1) and (2) is 510 – 465 = 54 parts.

The number of days growing these 54 parts is 30 – 24 = 6 days.

So each day the grass grows 54 ÷ 6 = 9 parts.

So the original amount of grass is 510 – 9 × 30 = 240 parts.
(or 455 – 9 × 24 = 240 parts).
In our problem, the total number of growth days the grass growth is 8. So the amount of grass growth is 8 × 9 = 72.

The total amount of grass in 8 days is 240 + 72 = 312 parts.

Suppose no cow was sold. These 4 cows will eat 4 × 2 (parts) = 8 parts of the grass.

The total amount of grass is 312 + 8 = 320 parts.

320 ÷ 8 = 40 cows.

24. Solution: 27.75.
(99 – 4) ÷ (100 – 0) × (25 – 0) + 4 = 27.75,

25. Solution: 1100.

$$16\div[36\%\times\frac{10}{11}-(1-36\%-36\%\times\frac{10}{11})]=16\div(\frac{0.36\times 20}{11}-0.64)=16\div\frac{0.16}{11}=1100\,.$$

26. Solution: $14.

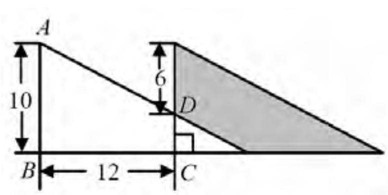
The price of the gift is $54\div(\frac{1}{\frac{3}{5}}+\frac{1}{\frac{3}{4}}+\frac{1}{\frac{2}{3}})=12$.

The sum of money remaining for Alex and Cindy is $12\div\frac{3}{5}+12\div\frac{2}{3}-12\times 2=\$14$.

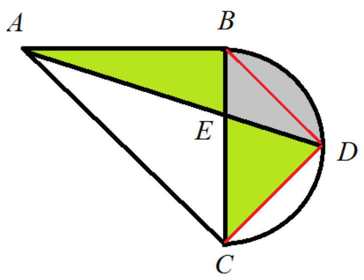
27. Solution:
The area of trapezoid *ABCD* is

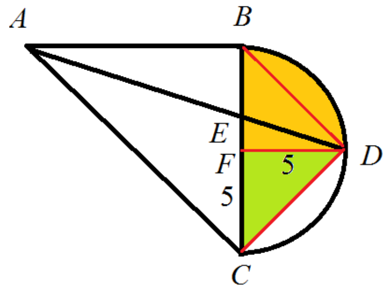
$$S=\frac{(10+4)\times 12}{2}=84$$

The shaded area is the same as the area of trapezoid *ABCD*.



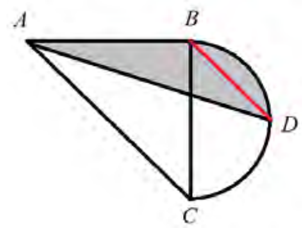
28. Solution: $\frac{1}{4}(50+25\pi)\text{ cm}^2$





Connect *BD* and *CD*. Since *BC* is the diameter, *BD* = *CD*. *BD* // *AC*.
So the area of triangle *ABE* is the same as the area of triangle *EDC*. We only need to find the sum of the areas of quarter circle *FDB* and right triangle *CDF*: $\frac{1}{4}(50+25\pi)\text{ cm}^2$.

Method 2:
Connect *BD*. The shaded area is



$$\frac{1}{2}AB\times(\frac{1}{2}BC)+[\frac{1}{4}\pi(\frac{1}{2}BC)^2-\frac{1}{2}(\frac{1}{2}BC)^2]$$
$$=\frac{1}{2}\times10\times5+\frac{1}{4}\pi\times25-\frac{1}{2}\times5^2$$
$$=\frac{1}{4}(50+25\pi)\text{ cm}^2.$$

29. Solution: 3:1.
Method 1:
Let the area of the circle C be 1.

$$S_A+S_B=\frac{2}{3}\quad\Rightarrow\quad\frac{1}{2}S_A+\frac{1}{2}S_B=\frac{1}{3}\qquad(1)$$

We also know that $\frac{1}{3}S_A+\frac{1}{2}S_B=\frac{1}{4}$ (2)

Solving the system of equations (1) and (2): $S_A=\frac{1}{2}$ and $S_B=\frac{1}{6}$. Thus $\frac{S_A}{S_B}=\frac{3}{1}=3:1$.

Method 2:
Let the area of the circle *A* be *a* and the area of the circle *B* be *b*..
We know that the sum of the arras of A and B is 2/3 of the area of the circle *C*.

The area of the circle C is $(a+b)\div\frac{2}{3}=\frac{3}{2}(a+b)$.

So $\frac{1}{3}a+\frac{1}{2}b=\frac{1}{4}\times\frac{3}{2}(a+b) \quad \Rightarrow \frac{1}{2}b-\frac{3}{8}b=\frac{3}{8}a-\frac{1}{3}a \quad \Rightarrow \quad \frac{1}{8}b=\frac{1}{24}a \Rightarrow \quad a:b=3:1$.

30. Solution: 11.

If the champion team won zero game, it is not possible for the team to get the highest points.

If the champion team won 1 game, other teams at least won two games. The team tied with the champion team got 7 points with at least 2 wins and 1 tie. The champion team at least got 8 points with 1 win, 5 ties. Other teams got 2 wins, at most 1 tie, and 3 losses. So we have 13 wins and at least 18 losses. This case is not possible.

Similarly we know that the champion team did not get 9 points or 10 points.

Suppose the champion team got 11 points with 2 wins and 5 ties and total number of teams is 8. Let 5 teams each get 3 wins, 3 losses and 1 tie. The rest of two teams each gets 3 wins and 4 losses. This case is possible. So at least the champion gets 11 points.

Solutions to Practice Test 15

1. Solution: 1111.
In each number, each of the digits 1, 2,3, and 4 appears exactly once in the units, tens digit, hundreds, and thousands digit, the answer is then
$(1234+2341+3412+4123)\div(1+2+3+4)=1111.$

2. Solution: 1936.
$44^2 = 1936$. $45^2 = 2025$. We know that $1936 < 2011 < 2025$. The greatest square number that is not exceeding 2011 is 1936.

3. Solution: 247.
The difference of the thousands digits is 1. In order to get the smallest difference, the last three digits of two numbers should be as far as possible.
876 and 123 give the greatest difference. So we have $5123 - 4876 = 247$.

4. Solution: 200.
Let w, b, and s be the cost for white horse, black horse, and the saddle, respectively.
$w + s = 800$ (1)
$b + s = 600$ (2)
$w + b = 1000$ (3)
(1) + (2): $w + b + 2s = 1400$ (4)
Substituting (3) into (4): $1000 + 2s = 1400 \Rightarrow 2s = 400 \Rightarrow s = 200$.

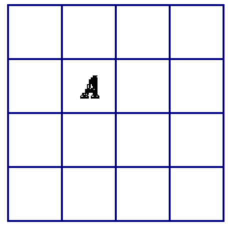
5. Solution: 15.
Since each monkey gets 8 bananas, 8 is the middle number. There are $7 + 1 + 7 = 15$ monkeys.

6. Solution: \$1160.
Method 1:
The number of type A buses needed: $(60 + 15) \div (60 - 45) = 5$.
The total number of students is $5 \times 45 + 15 = 240$.
Average cost for type A buses is $215 \div 45 = \$4.8$.
Average cost for type B buses is $300 \div 60 = \$5$.
So renting 4 type A buses and one type B bus will cost less.

The cost is $215 \times 4 + 300 = \$1160$.

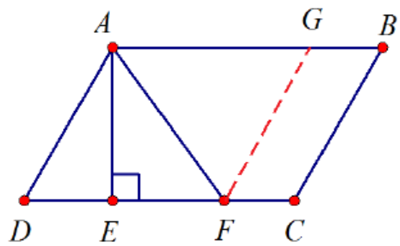
Method 2:
Let A and B be the number of buses needed for type A bus and type B bus, respectively.
$45A + 60B = 240 \quad \Rightarrow \quad 3A + 4B = 16 \quad \Rightarrow \quad A = 4$ and $B = 1$.
The cost is $215 \times 4 + 300 = \$1160$.

7. Solution: 20.

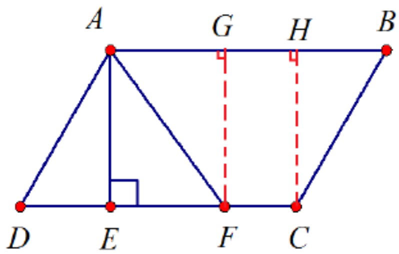


Total number of squares is $4^2 + 3^2 + 2^2 + 1^2 = 30$.
The number of squares containing A is $1 + 4 + 4 + 1 = 10$.
The answer is $30 - 10 = 20$.

8. Solution: 2 cm.
Method 1:



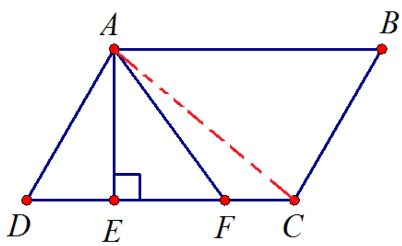
Draw $FG//CB$. The area of ΔADF is the same as the area of ΔAGF. So the area of parallelogram $GBCF$ is 14 cm^2.
Therefore $FC = 14 \div 7 = 2$ cm



Method 2:
Draw $GF \perp AB$ at G. $CH \perp AB$ at H. The area of ΔAEF is the same as the area of ΔAGF. The area of ΔADE is the same as the area of ΔCBH. So the area of rectangle $GHCF$ is 14 cm^2. Therefore $FC = 14 \div 7 = 2$ cm

Method 3:
Connect AC. The area of ΔABC is the same as the area of ΔADC. So the area of ΔAFC
$14 \div 2 = 7$ cm^2. So $\dfrac{FC \times AF}{2} = 7$

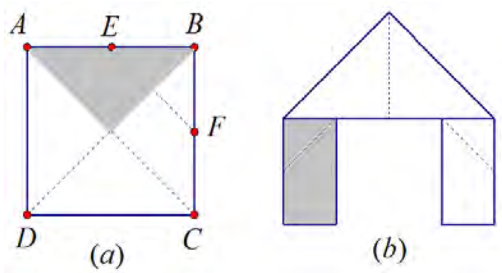
$\Rightarrow \qquad FC = 14 \div 7 = 2$ cm.



9. Solution: 102.
Method 1:
The number of pages Catherine has not read: $42 \div (2.4 - 1) = 30$
The number of pages Catherine has read: $30 \times 2.4 = 72$.
The answer is $72 + 30 = 102$.

Method 2:
Let x be the number of pages Catherine has read.
Let y be the number of pages Catherine has not read.
$x = 2.4y = 42 + y \quad \Rightarrow \quad y = 30$.
$x = 2.4y = 2.4y \times 30 = 72$.
The answer is $72 + 30 = 102$.

10. Solution: 100 cm^2.
The shaded area in figure (b) is $\frac{1}{4}$ of the area of the square $ABCD$.
So the answer is $\frac{1}{4} \times 20 \times 20 = 100 \text{ cm}^2$.



11. Solution: 8 minutes.
Method 1:
The total time for Bob, Charles, and Danny is $4 \times 3 = 12$.
The total time for Alex, Bob, Charles, and Danny is $4 \times 5 = 20$.
The time for Alex to run this 60-meter race is $20 - 12 = 8$ minutes.

Method 2:
$B + C + D = 12 \qquad (1)$
$A + B + C + D = 20 \qquad (2)$
(2) – (1): $A = 8$.

12. Solution: 156.

$345 \div 2 = 172.5$

$240 \div 2 = 120$

Great common multiple of 1725 and 1200 is 75.

So at most the distance between any two neighboring trees is 7.5 meters.

The number of trees planned is $(345 + 240) \times 2 \div 7.5 = 156$.

13. Solution: 1.04 m.

$9.77 \times 4 - 9.51 \times 4 = 1.04$ m.

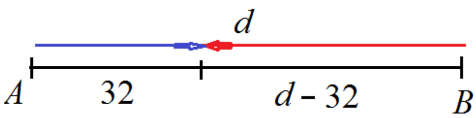
14. Solution: 80 km.

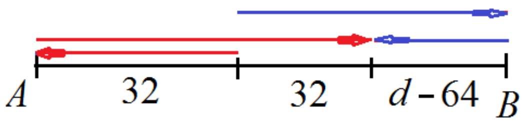
Method 1:

$(32 \times 3 + 64) \div 2 = 80$ km.

Method 2:

Let the distance from Ayden to Beaufort be d.



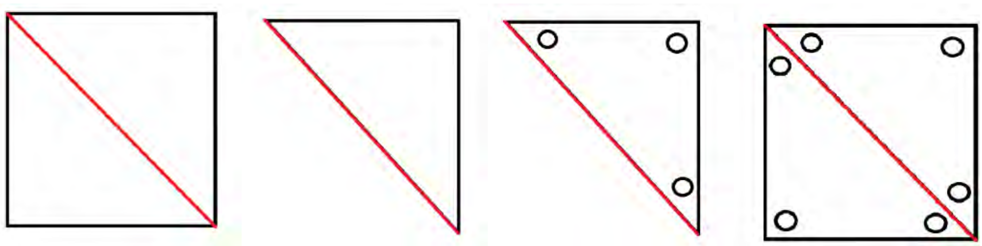


The ratio of the distances walked by Alex and Bob when they met first time is $\dfrac{32}{d-32}$

The ratio of the distances walked by Alex and Bob when they met second time is $\dfrac{32+2(d-64)}{32\times 3}$.

Since their speeds are constant, we have $\dfrac{32}{d-32} = \dfrac{32+2(d-64)}{32\times 3} \Rightarrow d = 80$ km.

15. Solution: C.

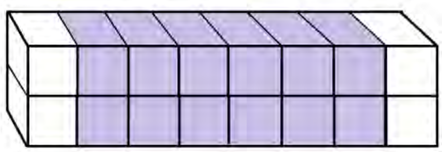


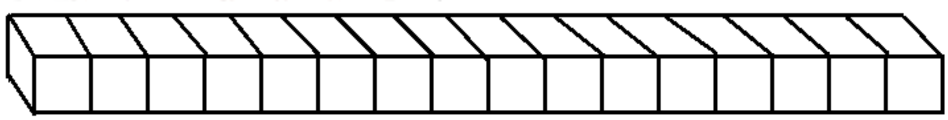
16. Solution: 100.
The numbers of cars sold for Toyota, Nissan and Honda are 40, 20, and 30, respectively.
The predicted total number of cars being sold in April will be
$40 \times 1.05 + 20 \times 1.1 + 30 \times 1.2 = 100$.

17. Solution: 11/12.
We know that $a \otimes b = \dfrac{m \times a + b}{2 \times a \times b}$ and $1 \otimes 4 = 2 \otimes 3$. So $\dfrac{m \times 1 + 4}{2 \times 1 \times 4} = \dfrac{m \times 2 + 3}{2 \times 2 \times 3} \Rightarrow m = 6$.
$3 \otimes 4 = \dfrac{m \times a + b}{2 \times a \times b} = \dfrac{6 \times 3 + 4}{2 \times 3 \times 4} = \dfrac{22}{24} = \dfrac{11}{12}$.

18. Solution: 501.
$$\frac{1}{\frac{1}{2005}+\frac{1}{2006}+\frac{1}{2007}+\frac{1}{2008}} > \frac{1}{\frac{1}{2005}\times 4} = \frac{2005}{4} = 501\frac{1}{4}$$
$$\frac{1}{\frac{1}{2005}+\frac{1}{2006}+\frac{1}{2007}+\frac{1}{2008}} < \frac{1}{\frac{1}{2008}\times 4} = \frac{2008}{4} = 502.$$
So the integer part is 501.

19. Solution: 12.
$16 = 1 \times 1 \times 16 = 1 \times 2 \times 8 = 1 \times 4 \times 4 = 2 \times 2 \times 4$.
The $1 \times 1 \times 16$ rectangular prism has 0 small cube painted three sides.
The $1 \times 2 \times 8$ rectangular prism has 12 small cube painted three sides.
The $1 \times 4 \times 4$ rectangular prism has 8 small cube painted three sides.
The $2 \times 2 \times 4$ rectangular prism has 8 small cube painted three sides.





The answer is $12 + 0 = 12$.

20、Solution: 1937.

$2010 = 2 \times 3 \times 5 \times 67$.

$\underbrace{1\times1\times1\times1\times1\times\cdots\times1}_{1933\ \ 1\text{'s}}\times2\times3\times5\times67=\underbrace{1+1+1+1+1+1+\cdots+1}_{1933\ \ 1\text{'s}}+2+3+5+67=2010$.

The greatest value of n is 1933 + 1 + 1 + 1 + 1 = 1937.

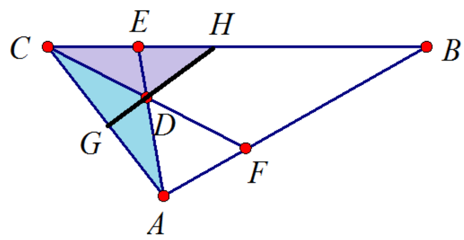
21. Solution: 1/2.

Method 1:

Draw $GH//AB$ through D.

The ratio of the areas of ΔACF and ΔCFB is the same as the ratio of the areas of ΔGDC and ΔDHC.

We know that $DF = DC$. So $CG = GA$ and the areas of ΔGDC and ΔGDA are the same.



We know that $AD = 2DE$. So the areas of ΔGDC, ΔGDA, and ΔDCE are the same.

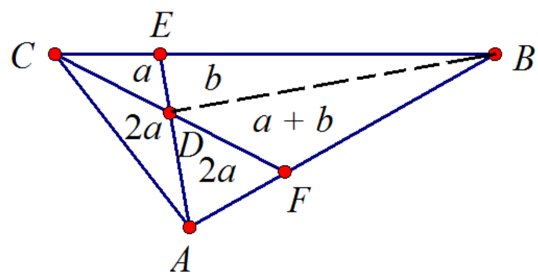
We know that $AD = 2DE$. So $2EH = BH$. We know that $DF = DC$. So $CH = BH$. So $CE = EH$. So the areas of ΔCED and ΔDEH are the same.

Thus the ratio of the areas of ΔGDC and ΔDHC is 1/2. The answer is 1/2.

Method 2:

Connect BD. Label the area of each region as shown in the figure. We know that $AD = 2DE$. So $2a + a + b = 2b \Rightarrow b = 3a$.

The ratio of the areas of ΔACF and ΔCFB is $2a/8a = 1/2$.



22. Solution: 8 seconds.

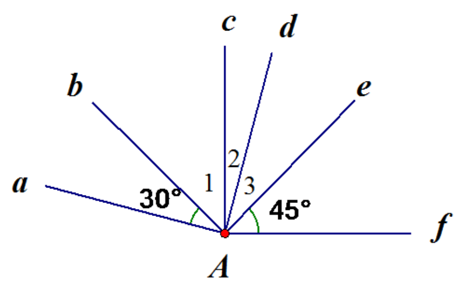
The sum of the speeds of two trains is 385 ÷ 11 = 35 meter/second.

$x = 280 \div 35 = 8$ seconds.

23. Solution: 15°.

As shown in the figure, $a \perp d$. So $\angle 1 + \angle 2 + 30° = 90°$ (1)

$b \perp e$. So $\angle 1 + \angle 2 + \angle 3 = 90°$ (2)
Thus $\angle 3 = 30°$.
$c \perp f$. So $\angle 2 + \angle 3 + 45° = 90° \Rightarrow \angle 2 = 90° - 45° - \angle 3 = 90° - 45° - 30° = 15°$.



24. Solution: 15.
We use the back calculation method.

	After opening	Before opening
Fifth day	0	$0 + 16 = 16$
Fourth day	$16 \div 2 = 8$	$8 + 16 = 24$
Third day	$24 \div 2 = 12$	$12 + 16 = 28$
Second day	$28 \div 2 = 14$	$14 + 16 = 30$
First day	$30 \div 2 = 15$	

25. Solution: 12 km.
Let a be the time Mr. Chan used to walk on a flat road, b be the time Mr. Chan used to walk uphill, and c be the time Mr. Chan used to walk downhill.

Since the distances are the same for the uphill road and the downhill road, $3b = 6c$, or $b = 2c$.
We know that $a + b + c = 3$
So $a + 2c + c = 3$ or $a + 3c = 3$.
The total distance he walks every day is
$4a + 3b + 6c = 4a + 3 \times 2c + 6c = 4a + 12c = 4(a + 3c) = 4 \times 3 = 12$ km.

26. Solution: 3:4:10.
Let the volume of the small-sized ball be 3 parts. Then the volume of the medium-sized is (3 + 1) parts. The volume of the large-sized ball is (4 + 6) parts.

The ratio is 3:4:10.

27. Solution: \$260.

We use the back calculation method. Let x be the amount of money for each person in the end.

	A	B	C	Total
After 3rd exchange	x	x	x	$3x$
After 2nd exchange	$x/2$	$2x$	$x/2$	$3x$
After 1st exchange	$x/4$	x	$(7/4)x$	$3x$
Starting	$(13/8)x$	$x/2$	$(7/8)x$	$3x$

Since Ana's money is \$100 in the end, $\frac{13x}{8} - x = 100$ $\Rightarrow$ $x = 160$.

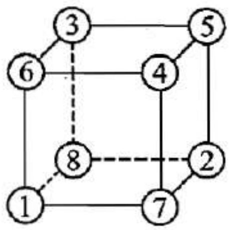
So $\frac{13x}{8} 160 \times \frac{13}{8} = 260$.

28. Solution: 21.

If we sum all the numbers of six faces, each number is added 3 times. So 3(1 + 2 + 3 + …+8) = 108.

The sum of 3 numbers on each face is then 108 ÷ 6 = 18.

The greatest possible sum of three numbers that are in the circles that are connected by the line segments to the circle with a "1" in it is 6 + 7+ 8 = 21.



29. Solution: 0.9 (hours).

Method 1:

Let m be the volume of water originally in the reservoir in m^3, and r be the rate of each pump.

We have

$m + 40 \times 2.5 = 5r \times 2.5$ (1)

$m + 40 \times 1.5 = 8r \times 1.5$ (2)

$m + 40 \times x = 13r \times x$ (3)

(1) – (2): $40 = 0.5r \quad \Rightarrow \quad r = 80\ m^3$/hour

Substituting $r = 80$ into (2): $m = 900\ m^3$.

Substituting the values of r and m into (3): $x = 0.9$ (hours)

Method 2:

The rate of each pump is 40 × (2.5 – 1.5) ÷ (5 ×2.5 – 8 × 1.5) = 80 m^3/hour

The volume of water originally in the reservoir is 80 × 8 × 1.5 – 40 × 1.5 = 900 m^3.

x, the time needed to pump out water from the reservoir with thirteen water pumps is then 900 ÷ (80 × 13 – 40) = 0.9 hours.

30. Solution:

Method 1:

Alex walks 1,200 meters longer than Bob in 100 minutes. So Alex walks 12 meters longer than Bob per minute.

After 10 minutes both of them are the same distance away from the intersection. So Bob walks (1,200 – 12 × 10) ÷ 10 ÷ 2 = 54 meters per minute.

After 100 minutes, Bob is 54 × 100 = 5,400 meters away from the intersection. So is Alex.

Method 2:

Let x be Alex's speed and y be Bob's speed.

We have

$1200 - 10x = 10y$ (1)

$100x - 1200 = 100y$ (2)

(1) × 10 + (2): $12000 - 100x + 100x - 1200 = 100y + 100y$.

So the distance Bob is from the intersection is 100y = (12000 – 1200) ÷ 2 = 5400 meters. Alex is 5400 meters from the intersection as well.

Solutions to Practice Test 16

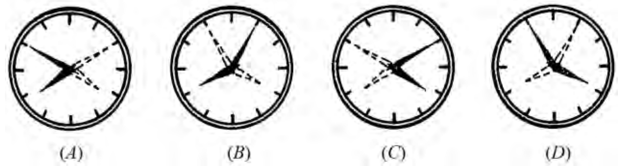
1. Solution: – 68.3.

$$2^3-\{(-3)^4-[(-1)\div 2.5+2\frac{1}{4}\times(-4)]\div(24\frac{8}{15}-26\frac{8}{15})\}$$

$$= 8-[81-(-\frac{2}{5}-9)\div(-2)]=8-(81-4.7)=-68.3.$$

2. Solution: D.

We reverse the reflection to get the correct figure (left ⇒ right; right ⇒ left). The time will be 4:10; 3:55; 7:50, and 8:05 for the figures (*A*), (*B*), (*C*), and (*D*), respectively.

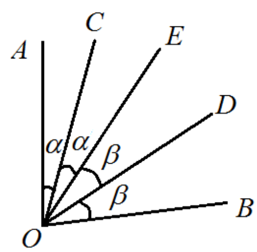


3. Solution: $\frac{1}{2}$.

Let x be the time in hours Sue needs to complete the job.

$$\frac{\frac{2}{5}}{1}=\frac{\frac{1}{5}}{x} \qquad \Rightarrow \qquad x=\frac{1}{2}.$$

4. Solution: $43°$.



We label each angle as follows.

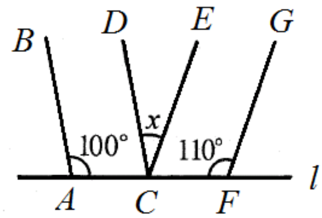
We know that $\alpha+\alpha+\beta+\beta=86° \quad \Rightarrow$

$2(\alpha+\beta)=86° \qquad \Rightarrow \alpha+\beta=43°= \angle COD.$

5. Solution: 0.

$$\frac{a^{17}+b^{18}+c^{19}}{a^{20}-b^{21}+c^{22}}=\frac{a^{17}+c^{19}}{a^{20}+c^{22}}=\frac{(-1)^{17}+1^{19}}{(-1)^{20}+1^{22}}=\frac{0}{2}=0.$$

6. Solution: 30°.

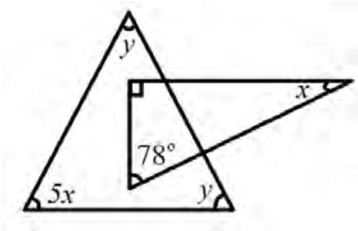


We know that $AB \parallel CD$, $CE \parallel FG$. So $\angle ACD = 180° - \angle BAC = 80°$. $\angle ECF = 180° - \angle GFC = 70°$.
So $\angle DCE = 180° - \angle ACD - \angle ECF = 180° - 80° - 70° = 30°$.

7. Solution: $-2x^2 + 6$.

$p_1(x) + p_2(x) - p_3(x) = 2x^2 + x - 3 + x^2 - 3x + 1 - (5x^2 - 2x - 8)$
$= -2x^2 + 6$

8. Solution: 60°.



$x + 78° = 90° \Rightarrow \quad x + 78° = 90° - 78° = 12°$.
$5x = 60°$.
$2y + 5x = 180° \Rightarrow \quad y = 60°$

9. Solution: -4.

$2x + 7 = 3 \quad \Rightarrow \quad x = -2$.
$b \times (-2) - 10 = -2 \quad \Rightarrow \quad b = -4$.

10. Solution: 27.

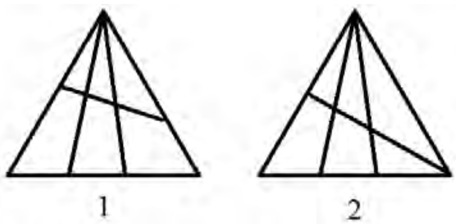


Figure 1:
Count by 1: 3
Count by 2: 5
Count by 3: 1
Count by 4: 2
Count by 6: 1
We get 12 triangles.
Figure 2:
Count by 1: 4
Count by 2: 5
Count by 3: 2
Count by 4: 2
Count by 6: 2

We get 15 triangles.
Then $m + n = 12 + 15 = 27$.

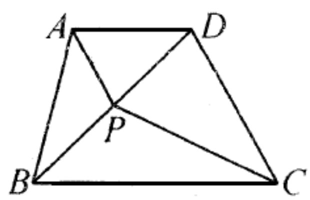
11. Solution: Tuesday
We got 366 days in 2008, 365 days in each of 2009 and 2010. And 77 days from January 1 to Match 18, 2011.

$366 + 365 + 365 + 77 = 4 \bmod 7$.

Day	Remainder
Friday	0
Saturday	1
Sunday	2
Monday	3
Tuesday	4
Wednesday	5
Thursday	6

The answer is: Tuesday.

12. Solution: 70.
We know that $S_{ABCD} = S_{\Delta APD} + S_{\Delta BPC} + S_{\Delta ABP} + S_{\Delta CDP}$.
We know that $S_{\Delta APD} = S_{\Delta ABP}$ (they have the same base and the same height) and $S_{\Delta BPC} = S_{\Delta CDP}$.
So $S_{ABCD} = 2(S_{\Delta APD} + S_{\Delta BPC}) = 2(15 + 20) = 70$.



13. Solution: 1.
Method 1:
The given equation can be written as $|x+3| - |x-1| = 2$.
We discuss the three intervals:
(1) $x > 1$.
The equation becomes: $x + 3 - (x - 1) = 2 \quad \Rightarrow \quad 4 = 2$ (no solutions).

(2) $1 \le x \le -3$.

The equation becomes: $x+3-(1-x)=2 \quad \Rightarrow \quad x=0$.

(3) $x < -3$.

The equation becomes: $-(x+3)-(1-x)=2 \quad \Rightarrow \quad -4 = 2$ (no solutions).

So the only solution is $x = 0$.

Method 2:

$|x+3|-|x-1|=2$ represents the difference of the distances from x to -3 and the distances from x to 1 is 2 . The only value for x is 0.

14. Solution: $a(1 + m\%) \times n\%$.

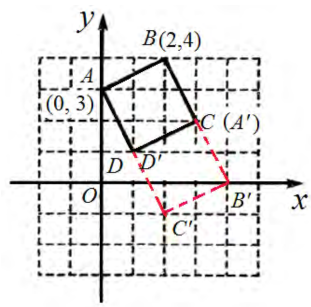
The first marked price is $a(1 + m\%)$. The final price is $a(1 + m\%) \times n\%$.

15. Solution: (A).

The area of the large square is $2^2 = 4$ and the area of the small square is $1^2 = 1$. When t = 0 or $t = t_0$, the shaded area is the same as the area of the large square, which is 4. When the small square is totally inside the large square, the shaded area is the smallest, which is $4 - 1 = 3$. (A) is the only answer.

16. Solution: 4.

When the square $ABCD$ is rotated 90° along the point D in a clockwise direction to become square $ABCD$, B is overlapped with B' (4,0). The sum of the coordinates is $4 + 0 = 4$.



17. Solution: 41.

Let the three prime numbers be a, b, and c.

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{551}{2431} \quad \Rightarrow \quad \frac{ab+bc+ca}{abc}=\frac{551}{2431}.$$

We see that $2431 = 11 \times 13 \times 17$ and $11 \times 13 + 13 \times 17 + 17 \times 11 = 551$.

So $a = 11$, $b = 13$, and $c = 17$.

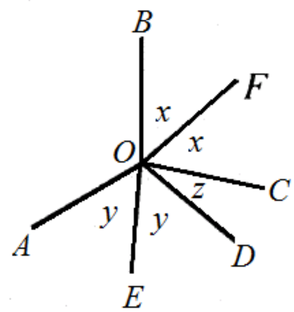
The answer is $11 + 13 + 17 = 41$.

18. Solution: – 8.

Let $x = 1$ and $y = 1$. We get the sum of the coefficients

$(2x^2 - 7xy + 3y^2)^3 = [2\times(1)^2 - 7\times1\times1 + 3\times(1)^2]^3 = (2-7+3)^3(-2)^3 = -8$.

19. Solution: 30°.



We label x, y, z as shown in the figure.

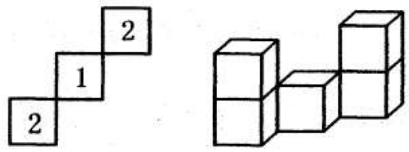
$2x + 2y + z = 360 - 120 \quad \Rightarrow \quad 2x + 2y + z = 240 \qquad (1)$

$x + y + z = 135 \qquad (2)$

(2) × 1 – (1): $z = 30°$.

20. Solution: 5.

We need at least 5 cubes.



21. Solution: 2011.

$x^2 - y^2 + x + y + 31$

$= (x^2 + x) - (y^2 - y) + 31$

$= x(x+1) - y(y-1) + 31$

= 3540 – 1560 + 31 = 2011.

22. Solution: (2, 3, 5)

(2) – (1): $3x - 2y = 0 \Rightarrow \quad 3x = 2y$.

Since $2y$ is even, $3x$ must be even. We know that x is a prime number. So $x = 2$.

Then $y = 3$, and $z = 5$.

23. Solution: 32.

Let x be the number of eggs laid by 6 hens in 8 days

$\dfrac{1.5}{1.5\times1.5} = \dfrac{x}{6\times8} \quad \Rightarrow \quad x = 32.$

24. Solution: 8.

We add three equations together: $a^2+b^2+c^2+2ab+2bc+2ca=64$ $\Rightarrow$ $(a+b+c)^2=64$ $\Rightarrow$ $a+b+c=8$ or $a+b+c=-8$ (ignored since it is negative). So the answer is 8.

25. Solution: 0.

$$\frac{2009!}{2008!}+\sum_{k=1}^{2008}k-\sum_{k=1}^{2009}k$$

$$=\frac{2009\times 2008!}{2008!}+(1+2+3+\cdots+2008)-(1+2+3+\cdots+2009)$$

$$=2009+(1+2+3+\cdots+2008)-(1+2+3+\cdots+2008)-2009$$

$= 2009 - 2009 = 0$.

26. Solution: 31.

We know that both a and b are positive integer. So we have $2a = 15 - 3b > 0 \Rightarrow b < 5$. Since $2a$ is even, $15 - 3b$ is even. So $3b$ is odd and b is odd. So we have $b = 1$ or $b = 3$. Then $a = 6$ or $a = 3$.

When $a = 6$ and $b = 1$, $a^2-ab+b^2=31$

When $a = 3$ and $b = 3$, $a^2-ab+b^2=9$. So the answer is 31.

27. Solution: 66.

Let the 11 consecutive positive integers be $x - 5, x - 4, x - 3, x - 2, x - 1, x, x + 1, x + 2, x + 3, x + 4, x + 5$.

Since their sum is 363, the middle number is $x=\frac{363}{11}=33$.

The smallest number is $33 - 5 = 28$. The greatest number is $33 + 5 = 38$.

The answer is $28 + 38 = 66$.

28. Solution: 4022.

In order to get the smallest value fro $2011(b-a)$, b must be as small as possible and a needs to be as large as possible. So we select that a = 2008 and b = 2010. The answer is $2011(2010-2008)=2011\times 2=4022$

29. Solution: 1.
We have
$b+c=ak$
$a+c=bk$
$a+b=ck$
We add them together: $2(a+b+c)=k(a+b+c)$ (1)
If $a+b+c\neq 0$, we solve (1) by dividing both sides by $a+b+c$: $k = 2$.
If $a+b+c=0$, we get $a+b=-c$.
$\frac{b+c}{a}=\frac{c+a}{b}=\frac{a+b}{c}=k \quad \Rightarrow \quad \frac{a+b}{c}=k \quad \Rightarrow \quad \frac{-c}{c}=k$.
So $k = -1$.
The sum of them is 1.

30. Solution: 1482.
$\frac{1}{A}-\frac{1}{B}=\frac{1}{988} \quad \Rightarrow \quad \frac{1}{A}=\frac{1}{988}+\frac{1}{B}=\frac{988+B}{988B} \quad \Rightarrow \quad A=\frac{988B}{988+B}$ or
$A=988-\frac{988\times 988}{988+B}$.
We know that A is positive integer. So 988 + B must be a factor of 988 × 988.
We know that $988 = 2^2 \times 13 \times 19$ which has (2 + 1)(1 + 1)(1 + 1) = 12 factors. Among these factors, we see 3 factors (1352, 1444, 1976) will result 3-digit number for both *A* (266, 312, 494) and *B* (364, 456, 988). The answer is 494 + 988 = 1482.

Solutions to Practice Test 17

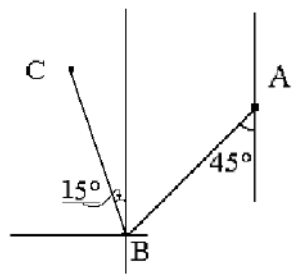
1. Solution: C.
We see a rectangle from the side view of (A) and (B). We see a square from the top view for (D). The answer is C.

2. Solution: – 192.
105 = 1 × 3 × 5 × 7.
The negative factors are –1, –3, –5, –7, –15, –21, –35, –105.
The sum is –(1 + 3 + 5 + 7 + 15 + 21 + 35 + 105) = –192.

3. Solution: (D).

4. Solution: 10.
At most there are $\binom{5}{2}=10$ points of intersection. The answer is 10.

5. Solution: 40.
$\angle ABC$ = 45° + 15° = 60°. So ΔABC is an equilateral triangle with AC = AB = 40.



6. Solution: $\frac{31}{32}$.

$$[-\frac{7}{5}\times(-2\frac{1}{2})-1]\div 9\div\frac{1}{(-0.75)^2}-\left|2+(-\frac{1}{2})^3\times 5^2\right|$$

$$= [-\frac{7}{5}\times(-\frac{5}{2})-1]\times\frac{1}{9}\times(\frac{3}{4})^2-\left|2+(-\frac{1}{8})\times 25\right|$$

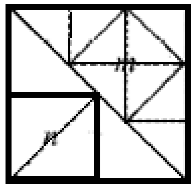
$$= (\frac{7}{2}-1)\times\frac{1}{9}\times\frac{9}{16}-\left|2-\frac{25}{8}\right| = \frac{5}{2}\times\frac{1}{9}\times\frac{9}{16}-\frac{9}{8}=\frac{5}{32}-\frac{9}{8}=-\frac{31}{32}.$$

So the answer is $(-1)\times(-\frac{31}{32})=\frac{31}{32}$.

7. Solution: 32.

$$\frac{12.5\times 26\%+15.8\times 29\%+10.2\times 44\%}{12.5+15.8+10.2}\times 100\%=\frac{12.32}{38.5}\times 100\%=32\%.\ m = 32.$$

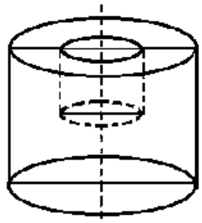
8. Solution: $\frac{9}{8}$.



Let the area of the large square be 1.

$m = \frac{1}{2}\times\frac{4}{9} = \frac{2}{9}$ and $n = \frac{1}{2}\times\frac{2}{4} = \frac{1}{4}$. So $\frac{m}{n} = \frac{\frac{1}{4}}{\frac{2}{9}} = \frac{1\times 9}{4\times 2} = \frac{9}{8}$.

9. Solution: 92π.



The resulting solid is the large cylinder with the small cylinder cut-off. The surface area is the surface area of the large cylinder plus the surface area of the small cylinder.

$\pi\times 8\times 6 + 2\times\pi\times 4^2 + \pi\times 4\times 3 = 92\pi$

10. Solution: 72.

Let g be the number of girls and $120 - g$ be the number of boys.

$\frac{g}{3} + \frac{120-g}{2} = 48$.

Solving for g we get: $g = 72$.

11. Solution: $-\frac{3}{2}$.

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{a} = k$, where $k \neq 0$.

$a = bk$ (1)

$b = ck$ (2)

$c = ak$ (3)

(1) × (2) × (3): $abc = abck^3 \quad\Rightarrow\quad k = 1$.

$\frac{3a+2b+c}{a-2b-3c} = \frac{3+2+1}{1-2-3} = -\frac{3}{2}$.

12. Solution: Bob.

Let the price for the first day be \$$x$/kg and second day be \$$y$/kg ($x \neq y$).

The average price for Alex: $\frac{10000x+10000y}{20000} = \frac{x+y}{2}$ (\$/kg)

The average price for Bob: $\dfrac{20000}{\dfrac{10000}{x}+\dfrac{10000}{y}}=\dfrac{2}{\dfrac{1}{x}+\dfrac{1}{y}}=\dfrac{2xy}{x+y}$ (\$/kg).

$\dfrac{x+y}{2}-\dfrac{2xy}{x+y}=\dfrac{(x+y)^2-4xy}{2(x+y)}=\dfrac{(x-y)^2}{2(x+y)}>0$. So Bob got a better deal.

13. Solution: 22.

Method 1:

Let the number of toy cars be x and the number of toy trucks be y. The number of toys be $m = x + y$.

$5x + 7y = 142$.

We know that $0 < x \le 28$ and $0 < y \le 20$.

$5x + 7y = 142 \Rightarrow 5x + 5y + 2y = 142 \Rightarrow 5m + 2y = 142 \Rightarrow$

$$m=\frac{142-2y}{5}.$$

We want the smallest value for m. So y should be as great as possible and $142 - 2y$ must also be divisible by 5. So $y = 16$ and $m = 22$.

Method 2:

$5x + 7y = 142 \Rightarrow 2y \equiv 2 \pmod 5 \Rightarrow y \equiv 1 \pmod 5$.

So y can be 1, 6, 11, or 16. The greatest value of y is 16. And $x = 6$. The answer is 16 + 6 = 22.

14. Solution: 9.

We see that $-\dfrac{2}{3}=-\dfrac{14}{21}$, and $-\dfrac{1}{3}=-\dfrac{7}{21}$. So we know that there are 6 fractions between $-\dfrac{2}{3}$ and $-\dfrac{1}{3}$.

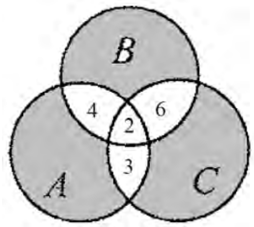
We see that $-\dfrac{2}{3}=-\dfrac{\frac{20}{3}}{10}$, and $-\dfrac{1}{3}=-\dfrac{\frac{10}{3}}{10}$. There are three integers between $\dfrac{20}{3}$ and $\dfrac{10}{3}$: 4, 5, and 6. So we know that there are 3 fractions with the denominator 10 between $-\dfrac{2}{3}$ and $-\dfrac{1}{3}$: $-\dfrac{4}{10}$, $-\dfrac{5}{10}$, $-\dfrac{6}{10}$. So the answer is 6 + 3 = 9.

15. Solution: $\dfrac{19}{4}$.

The length of *AB* is $\left|(-\dfrac{1}{2})-(-3)\right|=\dfrac{5}{2}$. The number marked by *A'* is $3-\dfrac{1}{2}\times\dfrac{5}{2}=\dfrac{7}{4}$.

The distance A moves: $\dfrac{7}{4}-(-3)=\dfrac{19}{4}$.

16. Solution: 58.



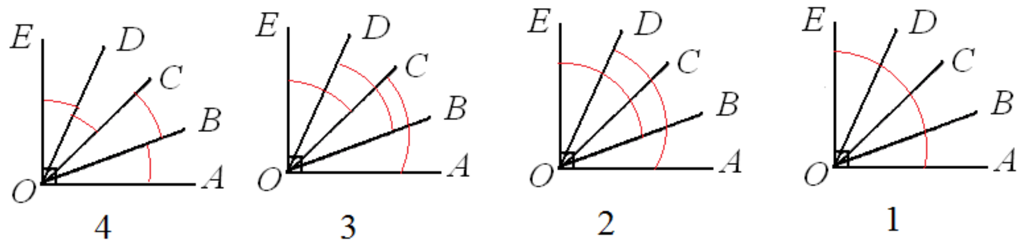
73 = 30 + 30 + 30 – 6 – 8 – 5 + x $\Rightarrow$ 6 – 8 – 5 + x = 2.

The shaded area is 73 – 6 – 8 – 5 + 2 × 2 = 58.

17. Solution: 450°.

We assume that four small angles are the same. Total we have $\binom{5}{2}=10$ angles not greater than 90°.

We count 4 angles of 45°/2, 3 angles of 45°, 2 angles of 45° + 45°/2, and 1 angle of 90°.



The sum is $4\times\dfrac{45°}{2}+3\times(45°)+2\times(45°+\dfrac{45°}{2})+1\times 90°=450°$.

18. Solution: 446.

$\dfrac{6}{7}=0.\overline{857142}$. The sum of the digits is 8 + 5 + 7 + 1 + 4 + 2 = 27.

2011 = 74 × 27 + 13.

The number of digits will be 74 × 6 = 444 that sum up to 2011 – 13 = 1998.

1998 + 8 + 5 = 2011. So the answer is 444 + 2 = 446.

19. Solution: (C).

$(a+b)^2-(a-b)^2=(a^2+2ab+b^2)-(a^2-2ab+b^2)=4ab$.

So $4ab = 4 \quad \Rightarrow \quad ab = 1$.
(C) is the correct answer.

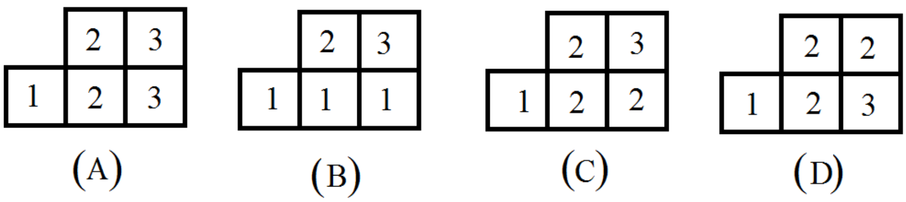
20. Solution: $-\frac{23}{72}$

$(x-2)^2 - 2(2-2x) - (1+x)(1-x)$

$= x^2 - 4x + 4 - 4 + 4x - 1 + x^2 = 2x^2 - 1 \quad x^2 - 4x + 4 - 4 + 4x - 1 + x^2 = 2x^2 - 1$

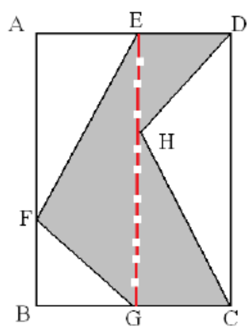
When $x = -\frac{7}{12}$, $2x^2 - 1 = 2 \times (-\frac{7}{12})^2 - 1 = -\frac{23}{72}$.

21. Solution: (B).
We view each solid from the top and number of the maximum possible number of cubes in the corresponding position.



For figure (A), we get 2 + 3 + 1 + 2 + 3 = 11 cubes.
For figure (B), we get 2 + 3 + 1 + 1 + 1 = 8 cubes.
For figure (C), we get 2 + 3 + 1 + 2 + 2 = 10 cubes.
For figure (D), we get 2 + 2 + 1 + 2 + 3 = 10 cubes.
Since (B) has no more than 8 cubes, (B) is not possible,

22. Solution: 12.



Connect *EG*. The area of the rectangle *ABCD* is $(2\times 3)\times(2\times 2) = 24$.

Since *E*, *H*, and *G* are collinear, $S_{\Delta EFG} = \frac{1}{2} S_{ABGE}$, and

$S_{\Delta CDH} = \frac{1}{2} S_{CDEG}$.

So the shaded area is $\frac{1}{2} S_{ABCD} = 12$.

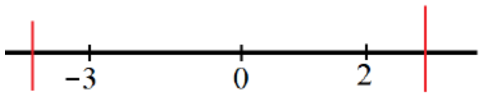
23. Solution: 90°.

$x + 2x + 3x = 180°$ $\Rightarrow$ $x = 30°$ $\Rightarrow$ $3x = 90°$.

24. Solution: 2.

Method 1:

The distance from x to 2 and –3 is 6. x will have two values and one is on the left of –3 and one is on the right of 2.



Method 2:

$|x-2|+|x+3|=6$ $\Rightarrow$ $|x-2|=6-|x+3|$.

So we get $x-2=6-|x+3|$ (1) or

$x-2=-(6-|x+3|)$ (2)

For (1), we have $|x+3|=8-x$.

$x+3=8-x$ $\Rightarrow$ $x=\frac{5}{2}$.

$x+3=-(8-x)$ (no solution).

For (2), we have $|x+3|=x+4$.

$x+3=x+4$ (no solution).

$x+3=-(x+4)$ $\Rightarrow$ $x=-\frac{7}{2}$.

We checked and both $x=\frac{5}{2}$ and $x=-\frac{7}{2}$ are the solutions.

25. Solution: D.

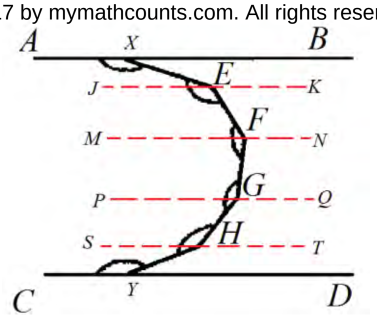
We see that $-|a|^3+b^3\ge 0$ $\Rightarrow$ $b^3\ge |a|^3$ $\Rightarrow$ $b\ge a$. The answer is (D).

26. Solution: 900°.

We draw *JEK// MFN // PGQ // SHT // CYD.*

$\angle XEJ + \angle AXE = 180°$ (1)

$\angle JEF + \angle MFE = 180°$ (2)



$\angle MHG + \angle PGF = 180°$ (3)

$\angle PGH + \angle SHG = 180°$ (4)

$\angle SHY + \angle DYH = 180°$ (5)

(1) + (2) + (3) + (4) + (5): $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 5 \times 180° = 900°$.

27. Solution: $\frac{5}{3}$.

Since $x = 0.7$ is the solution, $0.7a + \frac{1}{2} = \frac{5}{3} \quad \Rightarrow \quad a = \frac{\frac{5}{3} - \frac{1}{2}}{0.7} = \frac{7}{6} \times \frac{10}{7} = \frac{5}{3}$.

28. Solution: 60.

Let x be the number of parts machine A can make in one hour. $x + 8$ will be the number of parts machine B can make.

The number of hours machine A used is $\frac{\frac{1}{2}n}{x}$.

The number of hours machine B used is $\frac{\frac{1}{2}n}{x+8}$.

Since the total number of hours is 4, we have $\frac{\frac{1}{2}n}{x} + \frac{\frac{1}{2}n}{x+8} = 4 \Rightarrow \quad \frac{n}{x} + \frac{n}{x+8} = 8$ (1)

We know that the number of hours machine A used is $\frac{\frac{1}{2}n}{x}$ which is over 2 hours. So in 2 hours machine A made $2x$ parts.

In next two hours, machine A still worked $\frac{\frac{1}{2}n}{x} - 2$ hours so it made $x(\frac{\frac{1}{2}n}{x} - 2)$ parts.

Machine B worked $\frac{\frac{1}{2}n}{x+8}$ or $4 - \frac{\frac{1}{2}n}{x}$ hours and it made $(x+8) \times (4 - \frac{\frac{1}{2}n}{x})$ parts.

So we have $2x+12=x(\frac{\frac{1}{2}n}{x}-2)+(x+8)\times(4-\frac{\frac{1}{2}n}{x})$ $\Rightarrow$ $x=\frac{n}{5}$ (2)

Substituting (2) into (1): $\frac{n}{\frac{n}{5}}+\frac{n}{\frac{n}{5}+8}=8$ $\Rightarrow$ $\frac{n}{\frac{n}{5}+8}=3$ $\Rightarrow$ $n=3(\frac{n}{5}+8)$

$\Rightarrow$ $n=60$.

29. Solution: 18.

$x^4+7x^3+8x^2-13x+15$

$= x^2(x^2+2x)+5x(x^2+2x)-2(x^2+2x)-9x+15$

$= 3x^2+15x-6-9x+15 = 3(x^2+2x)+9=3\times 3+9=18$

30. Solution: 6641.

$xy=4\times 20=80$.

$x^2y^2+3xy+1=80^2+3\times 80+1=6641$.

Solutions to Practice Test 18

1. Solution: 22%.
The percent of the students participate in the Math Club is
100% – 17% – 26% – 35% = 22%.

2. Solution: 3.

We solve the inequality: $4 \le \frac{3x+7}{7} < 5$. X = 7, 8, and 9. The answer is 3.

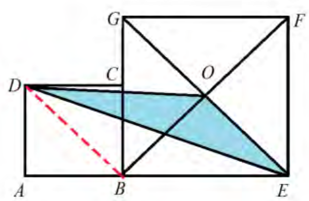
3. Solution: 15.

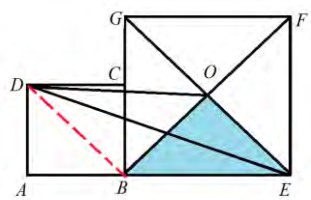
The number of positive integer solutions is $\binom{n-1}{r-1} = \binom{7-1}{3-1} = \binom{6}{2} = 15$.

4. Solution: $\frac{25}{4}$.

$S_{ABCD} = AB \times BC = 9 \quad \Rightarrow \qquad$ BC = 3.
So BG = BC + CG = 3 + 2 = 5.
$S_{BEGF} = BG^2 = 25$

Connect *BD*. We see that *BD* // *EG*. So ΔEDO and ΔBEO has the same area.
Thus $S_{\Delta BEO} = \frac{1}{4} S_{BEFG} = \frac{1}{4} \times 25 = \frac{25}{4}$





5. Solution: 351 km.
Alex used 3 × 60 ÷ 4 = 45 seconds to make one cartwheel. In 45 seconds, the rocket travelled 7.8 × 3 × 60 ÷ 4 = 351 km.

6. Solution: 50.

$a^2b + ab^2 = ab(a+b) = -30 \quad \Rightarrow \qquad 3ab = -30 \qquad\qquad \Rightarrow \qquad ab = -10$

$a^2 - ab + b^2 + 11 = (a^2 + 2ab + b^2) - 3ab + 11 = (a+b)^2 - 3ab + 11$

$= 3^2 - 3 \times (-10) + 11 = 9 + 30 + 11 = 50$.

7. Solution: 2004.

Profit rate for 2003: $\frac{300}{3000} \times 100\% = 10\%$

Profit rate for 2004: $\frac{360}{3200} \times 100\% = 11.25\%$

Profit rate for 2005: $\frac{480}{5000} \times 100\% = 9.6\%$.

The answer is 2004.

8. Solution:

$$\frac{13 \times 17 \times (-\frac{2}{13} + 0.125) \div (-1\frac{1}{16})}{1 - \frac{1}{2} - \frac{1}{8}} = \frac{17 \times (-2 + \frac{13}{8}) \times (-\frac{16}{17})}{\frac{3}{8}} = \frac{(-\frac{3}{8}) \times (-16)}{\frac{3}{8}} = 16$$

9. Solution: 9/4.

$$(m-2)[-\frac{1}{4}(\frac{1}{m} + 2)] = 1 \quad \Rightarrow \quad (m-2)(\frac{1}{m} + 2) = -4 \quad \Rightarrow \quad 1 - \frac{2}{m} + 2m = 0$$

$$\Rightarrow \quad \frac{1}{m} - m = \frac{1}{2} \qquad (1)$$

Squaring both sides of (1): $\frac{1}{m^2} + m^2 = \frac{9}{4}$

10. Solution: 421.

Let $n + 20 = a^2$ and $n - 21 = b^2$ (a and b are integers).

$a^2 - b^2 = (a-b)(a+b) = 41$, $a^2 > b^2$.

We know that 41 is a prime number. So we have

$\boxed{\begin{cases} a-b=1 \\ a+b=41 \end{cases}}$ $\boxed{\begin{cases} a-b=41 \\ a+b=1 \end{cases}}$ $\boxed{\begin{cases} a-b=-1 \\ a+b=-41 \end{cases}}$ $\boxed{\begin{cases} a-b=-41 \\ a+b=-1 \end{cases}}$

Solving we get $a = 21$ or $a = -21$.

So $n + 20 = a^2 = 441 \quad \Rightarrow \quad n = 421$.

11. Solution: -4.

$\frac{1}{9}\{\frac{1}{6}[\frac{1}{3}(\frac{x+a}{2}+4)-7]+10\}=1 \quad\Rightarrow\quad \frac{1}{6}[\frac{1}{3}(\frac{x+a}{2}+4)-7]+10=9 \quad\Rightarrow$

$\frac{1}{6}[\frac{1}{3}(\frac{x+a}{2}+4)-7]=-1 \quad\Rightarrow\quad \frac{1}{3}(\frac{x+a}{2}+4)-7=-6 \quad\Rightarrow$

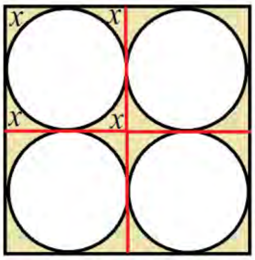
$\frac{x+a}{2}+4=3 \quad\Rightarrow\quad x+a=-2$

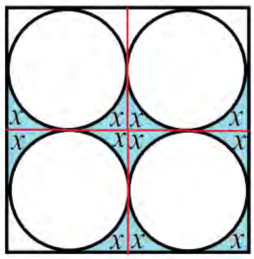
We are given that $x = 2$. So $x+a=-2 \quad\Rightarrow\quad a=-4$

12. Solution: 66.

$4 \times 4 - \pi \times 2^2 = 4x \quad\Rightarrow\quad x = 4-\pi$.

$11x = 11(4-\pi) = \frac{a}{7} \quad\Rightarrow\quad a = 66$.





13. Solution: \$1,415.

Let the original Price be x.

$x \times 0.75 \times 0.65 \times 0.6 = 585 \quad\Rightarrow\quad x \times \frac{3}{4} \times \frac{13}{20} \times \frac{3}{5} = 585 \quad\Rightarrow\quad x \times \frac{1}{4} \times \frac{1}{20} \times \frac{1}{5} = 5$

$\Rightarrow \quad x = 5 \times 5 \times 4 \times 20 = 100 \times 20 = 2{,}000$.

The saving is 2,000 – 585 = \$1,415.

14. Solution: 4.

Since $\overline{3434ab}$ is divisible by 9, 3 + 4 + 3 + 4 + a + b = 9 + 5 + a + b must be divisible by 9. So $a + b = 4$ or $a + b = 13$.

If $a + b = 13$, the last three digits of the six-digit number is 494, 485, or 476. None of them is divisible by 8.

$a + b$ must be 4. In fact 440 is divisible by 8 indeed. So $c = 4$.

15. Solution: 19.8.

$0.\overline{73}\times 1000b-0.73\times 1000b=146 \quad\Rightarrow\quad \frac{73}{99}\times 1000b-\frac{73}{100}\times 1000b=146 \quad\Rightarrow$

$1000b(\frac{73}{99}-\frac{73}{100})=146 \quad\Rightarrow\quad 1000b\times\frac{73}{9900}=146 \quad\Rightarrow$

$b=\frac{146\times 99}{730}=\frac{2\times 99}{10}=\frac{2(100-1)}{10}=20-0.2=19.8$.

16. Solution: 276.

Let the number of people who can speak both English and Spanish be x.

$2476 = 1765 + 987 - x \quad\Rightarrow\quad x = 276.$

17. Solution: $\frac{37}{64}$.

We see that $x\times(1-25\%)\times[y(1-25\%)]^2=\frac{27}{64}xy^2$.

So the percent decrease is $1-\frac{27}{64}=\frac{37}{64}$

18. Solution: 10°.

$2x + 30 \times 2 + 50 \times 2 = 180 \quad\Rightarrow\quad x = 10.$

19. Solution: 21.

8 cornered cubes are painted 3 faces each. 12 edged cubes are painted 2 faces each.

6 faced cube are painted 1 face each. Only one cube has no face painted.

The answer is 8 + 12 + 1 = 21.

20. Solution: 40.

The number of squares is $3 \times 4 + 2 \times 3 + 1 \times 2 = 20$.

The number of rectangles is $\binom{5}{2}\times\binom{4}{2}=10\times 6=60$

The answer is 60 – 20 =40.

21. Solution: 2

Method 1:

We know that $|(-2015)-(-1)| = 2014 = 503\times 4+2$. So – 2015 will meet the same number on the circle as – 3. So the number on the circle is 2.

Method 2:

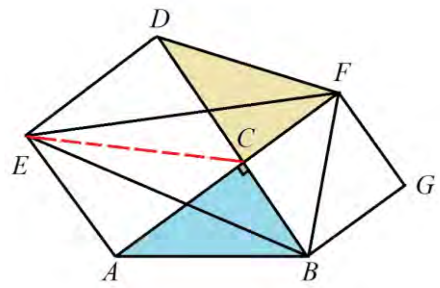
Number on the number line:	– 5	– 4	– 3	– 2	– 1
Number on the circle:	0	1	2	3	0.

$2015 \equiv 3 \mod 4$

So – 2015 and – 3 will be overlapping with the same number on the circle, which is 2.

22. Solution: 66.

Method 1:



$S_{\Delta ABC} = S_{\Delta CDE} = \frac{1}{2}\times 6\times 8 = 24$

$S_{BCFG} = 6^2 = 36$

$S_{AEDFGB} = 24+24+64+36 = 148$

Connect *CE*.

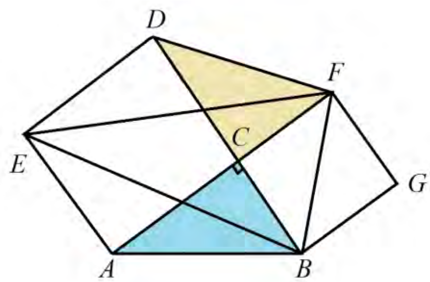
$S_{\Delta CFE} = S_{\Delta CFD} = 24$

$S_{\Delta CBE} = S_{\Delta CBA} = 24$

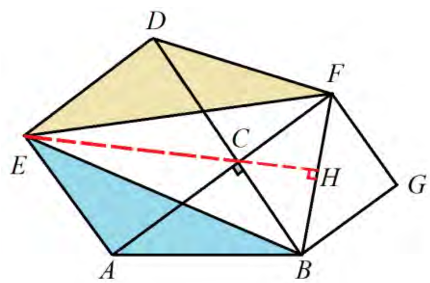
$S_{\Delta BCF} = \frac{6^2}{2} = 18$.

$S_{\Delta BEF} = 24+24+18 = 66$

Method 2:



$\Delta ABC \cong \Delta DFC$. So *DF* = *AB*. $\angle FDC = \angle BAC$. We also know that *DE* = *EA*. Thus $\Delta DEF \cong \Delta AEB$ and *EF* = *EB*. Triangle *BEF* is an isosceles triangle. Draw *EH* ⊥ *BF* at *H*. *H* is the center of square *CBG*F and *EH* goes through point *C*.



$EH = \sqrt{2}AC + \frac{\sqrt{2}BC}{2} = \sqrt{2}(8+\frac{6}{2}) = 11\sqrt{2}$.

$S_{\Delta BEF} = \frac{1}{2}FB\times EH = \frac{1}{2}\times\sqrt{2}\times 6\times 11\sqrt{2} = 66$.

23. Solution: 31.
Since b is divisible by both 4 and 3, it is divisible by 12.
Since b is also divides 60, b = 12 or 60.
Case 1: $b = 12$. $60 \div b = 5$.
We know that (5,4) = 1 and (5, 3) = 1. So at least one of a and c has the factor 5.
If a has a factor 5, then $a \ge 20$. Since $c \ge 3$, $a + b + c \ge 20 + 12 + 3 = 35$.
If c has a factor 5, then $c \ge 15$. Since $a \ge 4$, $a + b + c \ge 4 + 12 + 15 = 31$.
We see that a triangle can be formed when $a = 4$, $b = 12$, and $c = 15$.

Case 2: $b = 60$.
$a + b + c > 60 > 31$.
So the smallest value of $a + b + c$ is 31.

24. Solution: 0.
$2^{m+2011} - 3\times 2^m = 2^m(2^{2011} + 3)$.
The last digit of 2^{2011} is 2. So the last digit of $2^{2011} + 3$ is 5. So the last digit of $2^m(2^{2011} + 3)$ is 0.

25. Solution: 4.
DS, AX, EY, and *FZ* are parallel streets. So each of these streets needs a watchdog. So we need at least 4 watchdogs.

We can put one watchdog at *D, N, Y, F.* The answer is 4.

26. Solution: − 36.
$$m^3 - \frac{1}{m^3} = (m - \frac{1}{m})(m^2 + 1 + \frac{1}{m^2}) = 3(m^2 - 2 + \frac{1}{m^2} + 3)$$
$$= 3[(m-1)^2 + 3) = -3\times 12 = -36.$$

27. Solution: 45.
Let a, b, c, and d be the ages of Alex, Bob, Charlie, and Danny, respectively.
$$\frac{a+b+c}{3} + d = 29 \qquad (1)$$
$$\frac{b+c+d}{3} + a = 23 \qquad (2)$$

$$\frac{c+d+a}{3}+b=21 \qquad (3)$$

$$\frac{d+a+b}{3}+c=17 \qquad (4)$$

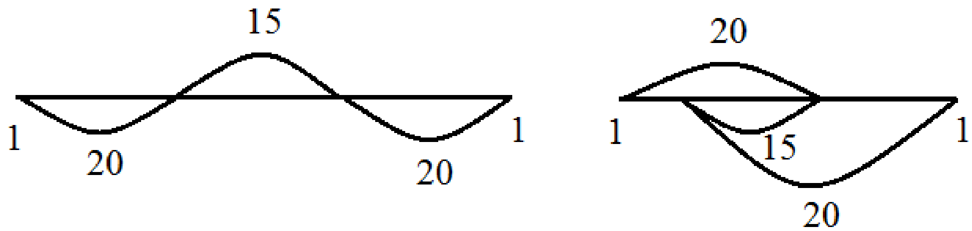
(1) + (2) + (3) + (4) : $a + b + c + d = 45$.

28. Solution: 78.

We have two cases as shown in the figure.

The number of kids in the class could be 20 + 20 + 13 = 53 or 20 + (20 - 15) = 25.

The answer is 53 + 25 = 78.



29. Solution: 1440.

Method 1:

Let t be the time Alex walked when he met Emily.

The time used for Alex walking from Ayden to Kinston is $(t - 4)$.

The time used for Emily walking from Kinston to Ayden is $(t - 5.5)$.

The time used for Emily walking from Ayden to Greenville is $(t - 2.5)$.

Let v be Alex's speed in meter per minute. Emily's speed is then $v + 30$ meters per minute.

$tv = (t-2.5)((v+30)$

$(t-4)v = (t-5.5)(v+30)$

Solving the system of equations we get: $t = 10$ and $v = 90$.

The distance from Greenville to Kinston is $(2t - 4)\ v = 16 \times 90 = 1440$ meters.

Method 2:

Let v be Alex's speed in meter per minute. Emily's speed is then $v + 30$ meters per minute.

$\overline{GA} - \overline{AK} = 4v$

$\overline{GA} - \overline{AK} = 3(v+30)$

So $v = 90$.

Let t be the time Emily walked when she met Alex.

$(5.5 + t) \times 90 - (90 + 30)t = 90 \times 4 \quad \Rightarrow \quad t = 4.5$.

The time Alex walked when he met Emily is then 5.5 + 4.5 = 10 minutes.
The distance from Greenville to Kinston is $90 \times 10 + (90 + 30) \times 4.5 = 1440$ meters.

30. Solution: 3122.
Let 9 numbers be *a, b, c, d, e, f, g, h*, and *i*.
The sum of all the possible 7-digit numbers is
$\overline{abc} + \overline{bcd} + \overline{cde} + \overline{def} + \overline{efg} + \overline{fgh} + \overline{ghi}$
$= 100a + 110b + 111(c + d + e + f + g) + 11h + i$.
In order to get the greatest possible sum, we let $a = 3$, $b = 4$, c to g be 5 to 9, $h = 2$ and $i = 1$.
The greatest sum is 4648.

Note the smallest sum will be 3122 ($a = 7$, $b = 6$, c to g be *1* to 5, $h = 8$, and $i = 9$).

Solutions to Practice Test 19

1. Solution: 15.

We list all the prime numbers less than 47: (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47). n is 15.

2. Solution: 16.

We see from (a) that the number of dots in two opposite faces: (4, 2); (6, 3), and (5, 1). So the sum of four numbers on the four bottom faces is 1 + 3 + 6 + 6 = 16.

3. Solution: 3.

$$\begin{vmatrix} 2x & -4 \\ x & 1 \end{vmatrix} = 2x - (-4)\,x = 18 \qquad \Rightarrow \qquad x = 3.$$

4. Solution: 5.

The number of matches Alex won is 20 × 95% = 19. Let x be the number of matches he needs to win in order to win 96% of his matches. Then

$$\frac{19+x}{20+x} = 96\% \qquad \Rightarrow \qquad 19 + x = \frac{96}{100}(20+x) \quad \Rightarrow \qquad 19 + x = 19.2 + 0.96x$$

$$\Rightarrow \qquad 0.04x = 0.2 \Rightarrow x = 5.$$

5. Solution: 16.

Applying Pythagorean Theorem to ΔABC and ΔADC:

$$AB^2 = m^2 - 5^2 = n^2 - 3^2 \qquad \Rightarrow \qquad m^2 - n^2 = 5^2 - 3^2 = 16$$

6. Solution: 8.

$$\frac{p+q+r}{3} = 4 \qquad \Rightarrow \qquad p + q + r = 12 \qquad (1)$$

$$\frac{p+q+r+x}{4} = 5 \qquad \Rightarrow \qquad p + q + r + x = 20 \qquad (2)$$

(2) – (1): $x = 8$.

7. Solution: $\frac{38}{3}$.

Let $a = 2k$, $b = 3k$, $c = 5k$ ($k \neq 0$)

From $a^2 + b^2 + c^2 = 4k^2 + 9^2 + 25k^2 = 38k^2$ and $abc = 30k^3$, we get

$30k^3 = 38k^2 \quad \Rightarrow \quad k = \frac{19}{15}$.

$a + b + c = 2k + 3k + 5k = 10k = 10 \times \frac{19}{15} = \frac{38}{3}$.

8. Solution: 10.

Method 1:

Let d be the circumference of the circle and x be the number of rounds Betsy runs:

$\frac{25}{60}[(\frac{(16+x)d}{15} + \frac{xd}{15}] = d$

Solve for x: $x = 10$.

Method 2:

Let V_a and V_b be the speeds of Alex and Betsy, respectively.

$$\frac{25}{60}(V_a + V_b) = d \qquad (1)$$

$$15(V_a - V_b) = 16d \qquad (2)$$

Solve for V_a/V_b: $\frac{V_a}{V_b} = \frac{13}{5}$.

We also know that $\frac{V_a}{V_B} = \frac{16+x}{x} \quad \Rightarrow \quad \frac{13}{5} = \frac{16}{x} + 1 \quad \Rightarrow \quad \frac{8}{5} = \frac{16}{x} \quad \Rightarrow \quad x = 10$.

9. Solution: 56.25.

100(1 – 15%) = (100 + x)(1 – 15% – 5%) $\Rightarrow$ x = 6.25 kg.

100 × 15% = (100 + y)(15% – 5%) $\Rightarrow$ y = 50 kg.

$x + y = 6.25 + 50 = 56.25$ kg.

10. Solution: 2.

$P = a + b + c = 25$. $\qquad c < P/2 = 12.5$.

12 is not a prime number. If $c = 11$, $a + b = 14$. We have $a = 3$, $b = 11$; or $a = 7$, $b = 7$.
If $c \le 7$, $a + b = 18$. It violates the condition that $a \le b \le c$.
So we can have only two solutions.

11. Solution: 5.

$ax + 4y = 8$ (1)

$3x + 2y = 6$ (2)

(2) × 2 − (1): $x = \dfrac{4}{6-a}$.

Since $x > 0$, we get $a < 6$.

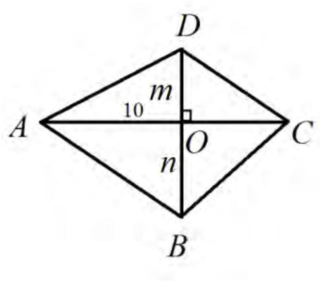
(2) × a − (1) × 3: $y = \dfrac{3a-12}{a-6}$.

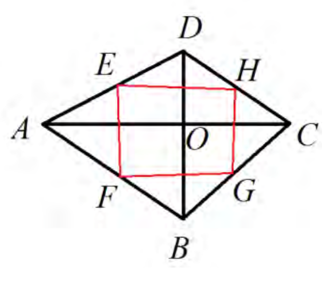
Since $y < 0$, we get $4 < a < 6$. The only positive integer is 5 for a.

12. Solution: 20.

The area of quadrilateral $ABCD$ is $\frac{1}{2}AC \times DO + \frac{1}{2}AC \times BO = \frac{1}{2}AC \times DB = 40$.

The area of quadrilateral $EFGH$ (parallelogram) is $\frac{1}{2} \times ABCD = \frac{40}{2} = 20$.





13. Solution: 1:1.

The side length of the square is $\dfrac{a}{\sqrt{2}}$ and the area is $\dfrac{a^2}{2}$.

The area of the rhombus is $\dfrac{bc}{2}$.

We know that $b:a = a:c \quad \Rightarrow \quad a^2 = bc \quad \Rightarrow \quad \dfrac{a^2}{2} = \dfrac{cb}{2}$.

So the ratio is 1: 1.

14. Solution: 132.
Since the triangle is a right triangle, one of the three sides must be divisible by 3; one of the three sides must be divisible by 4; one of the three sides must be divisible by 5.

So 11 must be the length of one of the two legs.

Let the length of the other leg be *b* and the length of the hypotenuse be *c*.
Then we have $11^2 + b^2 = c^2 \quad \Rightarrow \quad (c-b)(c+b) == 121$
We know that $c - b < c + b$. So we only have
$c - b = 1$ (1)
$c + b == 121$ (2)
Solving we get *b* = 60 and *c* = 61.
The perimeter is 11 + 60 + 61 = 132,

15. Solution: 18.
Solving the equation we get *x* = 6 or 3.
So we have the following triples (*a, b, c*): (6, 6, 6), (6, 6, 3), (3, 3, 3), and (3, 3, 6). Last triple will not form a triangle. So the greatest possible value of the perimeter of the triangle is 6 + 6 + 6 = 18.

16. Solution: 9.
The original equation can be written as $(10a+b)((b+1) = (10b+a)(a+1) \Rightarrow$
$10ab + b^2 + 10a + b = 10ab + a^2 + 10b + a \Rightarrow b^2 - a^2 = 9(b-a)$ (1)
We know that *a* and *b* are distinct real numbers. So *b* – *a* is not zero. Then we divide (1) by *b* – *a: b* + *a* = 9.

17. Solution: 27.
7*x* + 19 *y* = 213 $\Rightarrow$ 5*y* = 10 mod 7 $\Rightarrow$ *y* = 2 and *x* = 25.
The greatest value of *x* + *y* is 2 + 25 = 27.
(We can also get *y* = 9 and *x* = 6. *x* + *y* is 9 + 6 = 15).
18. Solution: 6.
We see that *x* = 0, 1, 2, and 3 are the integer solutions. So the answer is 6.

19. Solution: –1.

Since point (1, 2) is on the curve, we know that a = 2.

We also have $y = \frac{2}{x} = x + 1 = \frac{2}{b}$.

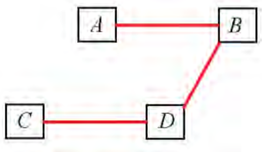
From $\frac{2}{x} = x + 1$, we get $x^2 + x - 1 = 0$. Solving for x: x = 1 or x = –2.

So b = 1 or b = – 2. The sum is –1.

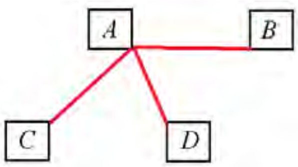
20. Solution: 16.

Case I: *ABDC* is one way. We have 4! = 24 ways. Note that *ABDC* is the same as *CDBA*. So we only get 24/2 = 12 ways.

s one way. one way.



Case II: We have 4 ways.



Total we get 12 + 4 = 16 ways.

21. Solution: 39.

$A + B + C = 55$ (1)

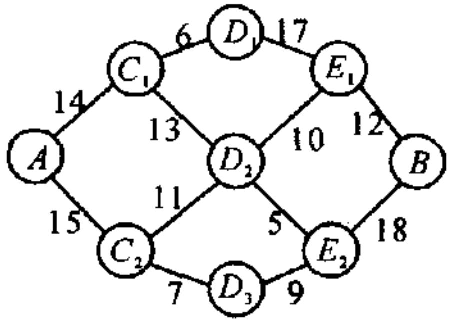
$A = 3B \quad \Rightarrow \quad B = A/3$ (2)

Substituting (2) into (1): $A + A/3 + C = 55$ (3)

$A = \frac{165 - 3C}{4}$. The greatest value of A is obtained when C = 3, which is 39.

22. Solution: 48.

Let the time needed to go from city x to city B is S.



$S(E_1) = 12$, $S(E_2) = 18$

$S(D_1) = 17 + S(E_1) = 29$

$S(D_2) = \min\{10 + S(E_1),\ 5 + S(E_2)\} = 22$

$S(D_3) = 9 + S(E_2) = 27$

$S(C_1) = \min\{6 + S(D_1),\ 13 + S(D_2)\} = 35$

$S(C_2) = \min\{11 + S(D_2), 7 + S(D_3)\} = 33$

$S(A) = \min\{14 + S(C_1), 15 + S(C_2)\} = 48$.

So the time needed ti 48 hours and the route is $A - C_2 - D_2 - E_1 - B$.

23. Solution: 31.

We use the indirect way.

The total number of ways to put five lids randomly on five cups is 5! = 120.

By the Derangement formula, the number of ways that no lid matches the cup with the same number is $D_n = n![1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^n \frac{1}{n!}]$.

The number of ways that all five lids mismatched is $D_4 = 5!(1 - \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!}) = 44$.

The number of ways that four of the five lid mismatched is

$5 \times D_4 = 5 \times 4!(1 - \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!}) = 45$.

The answer is 120 – 44 – 45 = 31.

24. Solution: $\frac{1}{5}$.

We use the indirect way.

There are 6! = 120 ways to seat 6 people.

We treat three girls as a unit and three boys as three units. So we need to arrange 4 units in a row. We have 4! Ways to do so. Since the girls can change the positions, so we get the number of ways that exactly three girls are seated together: 4! × 3!.

The probability is $P = \frac{4! \times 3!}{6!} = \frac{4! \times 3!}{6 \times 5 \times 4!} = \frac{6}{6 \times 5} = \frac{1}{5}$.

25. Solution: 5.
The pattern repeats every 16 terms when mod 7: 1, 1, 2, 3, 5, 1, 6, 0, 6, 6, 5, 4, 2, 6, 1, 0).
$2011 = 16 \times 265 + 11$. So 2011th term is the same as 11th term, which is 5.

26. Solution: 5.
We know from the shape of the time counter that the height decreases slowly at first and then decreases fast. So B is the answer.

27. Solution: 16.
We have two cases where the current goes from P to Q.
For figure a, if A and B are on, D can be on or off and C can be on or off. So we have 4 ways. Similarly we get 4 ways if D and E are on.

For figure b, if A, C, and E are on, D can be on or off and B can be on or off. So we have 4 ways. Similarly we get 4 ways if A and E are on.
The answer is $8 + 8 = 16$.

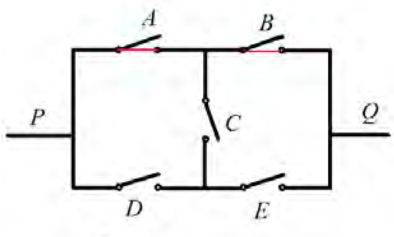


Figure a

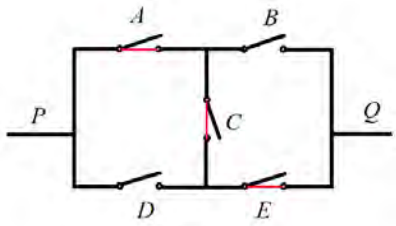


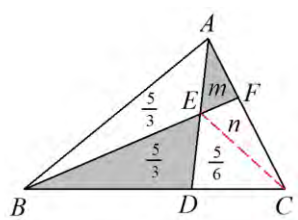
Figure b

28. Solution: 2.
Since triangle ABC has the area of 5, triangle ABD has the area of 10/3.

Since $AE = ED$, triangle ABE and triangle BDE has the same area of 5/3. Triangle ADC has the same area of 5/3 as well.
Connect CE. Let m and n be the areas of triangle AEF and triangle CEF, respectively.

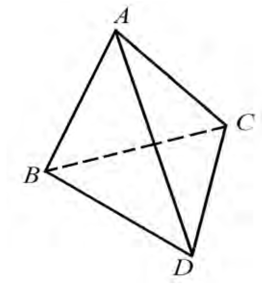
We have $m + n = \dfrac{5}{3}$ (1)

We have $\dfrac{\frac{5}{3}}{m} = \dfrac{\frac{5}{3} + \frac{5}{6}}{n} \Rightarrow 3m = 2n$ (2)



Solving (1) and (2) for *m*: $m = \frac{1}{3}$. The answer is $\frac{5}{3} + \frac{1}{3} = 2$.

29. Solution: $\frac{7}{27}$.



After travelling four edges, let us say *ACDBA*, the number of ways the ant will be home is $3 \times 3 \times 3 \times 3 = 81$.
The ant has three ways to start (from *A* to *C*, from *A* to *B* or from *A* to *D*).
When the ant is at *C*, it still have 3 ways to go (to *A*, to *B*, or to *D*).

Sub-cases:
(1) When the ant is at *D*, it has only two ways to go (C or B, but not A). Then it has one way to get home vertex.
(2) When the ant is at *B*, it has only two ways to go (C or D, but not A). Then it has one way to get home vertex.

(3) When the ant is at *A*, it has three ways to go (B, C or D). Then it has one way to get home vertex.
So we get 3(2 × 2 × 1 + 1 × 3 × 1) = 3 × 7.
Therefore the probability is $P = 3 \times 7 \times \frac{1}{81} = \frac{7}{27}$.

30. Solution: 4691.
The least common multiple of 2, 3, and 7 is 42. The number of terms left after removing in the set {1, 2, 3…,42} is

$$42 - [\left\lfloor \frac{42}{2} \right\rfloor + \left\lfloor \frac{42}{3} \right\rfloor + \left\lfloor \frac{42}{7} \right\rfloor - [\left\lfloor \frac{42}{2 \times 3} \right\rfloor + \left\lfloor \frac{42}{2 \times 7} \right\rfloor + \left\lfloor \frac{42}{3 \times 7} \right\rfloor - \left\lfloor \frac{42}{2 \times 3 \times 7} \right\rfloor + \left\lfloor \frac{42}{7} \right\rfloor] = 18.$$

Denote these terms as $\{a_n\}$: {1, 5, 7, 11, 13, 14, 17, 19, 21, 23, 25, 28, 29, 31, 35, 37, 41, 42}, where n = 1 to 18.

We see that 2011 = 111 × 18 + 13. So $a_{2011} = 111 \times 42 + a_{13}$ = 4462 + 29 = 4691.

Solutions to Practice Test 20

1. Solution: 1.

$2008\times2006 + 2007\times2005 - 2007\times2006 - 2008\times2005$

$= 2008\times(2006 - 2005) - 2007\times(2006 - 2005) = 2008 - 2007 = 1.$

2. Solution: 188.

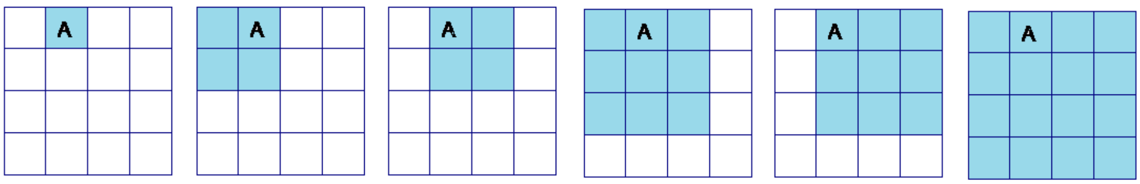
Since $m = 20n + 8$, we know that $n > 8$.

$m = 20n + 8 = 4(5n + 2)$. Since we want the smallest value of m, we let $n = 9$ and we get $m = 188$.

3. Solution: 24.

Total we get $4^2 + 3^2 + 2^2 + 1^2 = 30$ squares.

We count the number of squares containing the letter A:



The answer is $30 - 6 = 24$.

4. Solution: 240.

$$20y = x \qquad (1)$$

$$16(y + 3) = x \qquad (2)$$

Solving the system of equations (1) and (2): $x = 240$.

5. Solution: 10.

Let A and B be Alex's and Bob's ages, respectively.

$$A = \frac{1}{4}B \qquad (1)$$

$$A + 15 = \frac{5}{11}(B + 15) \qquad (2)$$

Solving for A: $A = 10$.

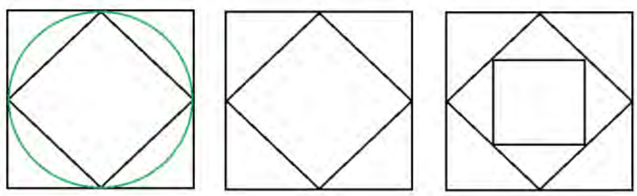
6. Solution: 86,400 seconds.

When the hour hand completes 1 revolution, the minute hand completes 12 revolutions, and the second hand completes $60 \times 12 = 720$ revolutions. The sum of the numbers of complete rotations is $1 + 12 + 720 = 733$.
Now the sum of the numbers of complete rotations is 1466. So the second hand completes $720 \times (1466 \div 733) = 1440$ revolutions, which takes $1440 \times 60 = 86{,}400$ seconds.

7. Solution: 900.
Bob walks $90 - 60 = 30$ meters more than Alex every minute.
When they meet, Alex is $60 \times 3 = 180$ meters from the school.
When they meet, Bob travelled $60 \times 3 \times 2 = 360$ meters more than the distance Alex travelled.
So when they meet, Alex's traveling time is $360 \div 30 = 12$ minutes.
The distance from Alex's home to his school IS $60 \times 12 + 180 = 900$ meters.

8. Solution: 50.
Since the perimeter of the rectangle *EFCB* is 100 cm, $EF + EB = 100 \div 2 = 50$
We know that $AE = EF$. So $AB = AE + EB = EF + EB = 50$ cm.

9. Solution: 15.
We rotate the second square 45°. We see that the area of the second square is half of the area of the largest square. The area of the smallest square is 1/2 of the area of the second square and is 1/4 of the area of the largest square. So the answer is $60 \div 4 = 15$.



10. Solution: B.
We see that A is next to B, C, D, and E. So A must be opposite to F.
We see that D is next to A, E, B, and E. So D must be opposite to C.
Then we know that B is opposite to the letter E.

11. Solution: 48.
The least common multiple of 2, 3, and 4 is 12. So 12 students can do 12 paper pigs, 6 clay pigs, 4 cotton pigs, and 3 metal pigs. Total 25 toy pigs.

Then the number of students is $100 \div 25 \times 12 = 48$.

12. Solution: $\frac{1}{21}$.

$$\frac{a-b\div c}{a+b\times c} = \frac{1-2\div 3}{1+2\times 3} = \frac{1-\frac{2}{3}}{7} = \frac{1}{3\times 7} = \frac{1}{21}.$$

13. Solution: 41.
Method 1:
$n - 1$ is divisible by 2, 4, and 5 or divisible by 20. So $n = 20k + 1$, where k is any positive integer.
$n + 1$ is divisible by 3. So $20k + 1+ 1$ is divisible by 3, or $20k + 2 = 18k + 2k + 2$ is divisible by 3, or $2k + 2$ is divisible by 3. Since we want to have the smallest value of n, we see that when $k = 2$, $2k + 2$ is divisible by 3. Then $n = 20k + 1 = 41$.

Method 2:

$$\boxed{\begin{array}{ll} n\equiv 1 & \bmod 2\\ n\equiv 2 & \bmod 3\\ n\equiv 1 & \bmod 4\\ n\equiv 1 & \bmod 5\end{array}} \Rightarrow \begin{array}{c}\boxed{\begin{array}{ll} n\equiv 1 & \bmod 2\\ n\equiv 1 & \bmod 4\\ n\equiv 1 & \bmod 5\end{array}}\\ \boxed{\begin{array}{ll} n\equiv 2 & \bmod 3\end{array}}\end{array} \Rightarrow \boxed{\begin{array}{ll} n\equiv 1 & \bmod 20\\ n\equiv 2 & \bmod 3\end{array}} \Rightarrow \boxed{n\equiv 41 \quad \bmod 60}$$

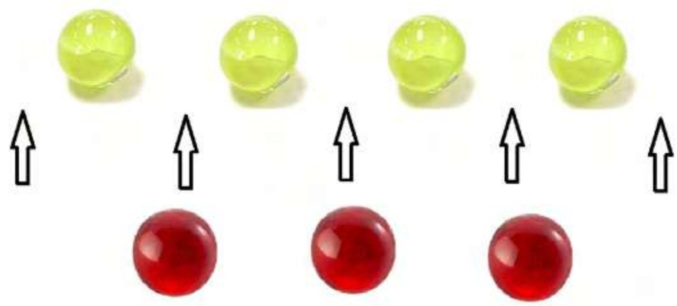
So the answer is 41.

14. Solution: 37.
Since both p and p^3+5 are prime numbers, p must be 2. $p^5 + 5 = 37$.

15. Solution: (4).
By observation, the answer is (4).

16. Solution: 10.

We have 5 places to put three red marbles. The number of ways is $\binom{5}{3}=10$.



17. Solution: 11.

We know that $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$. So $x+y+z=2+3+6=11$.

18. Solution: 8: 15.

$a:b=\frac{3}{2}:1.2=\frac{3}{2}:\frac{6}{5}=5:4=15:12$.

$b:c=0.75:\frac{1}{2}=\frac{3}{4}:\frac{1}{2}=3:2=12:8$

Thus $c:a=8:15$.

19. Solution: $\frac{2}{81}$.

$$\frac{\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{6}\right)\left(1-\frac{1}{7}\right)\left(1-\frac{1}{8}\right)\left(1-\frac{1}{9}\right)}{0.1+0.2+0.3+0.4+0.5+0.6+0.7+0.8+0.9}$$

$$=\frac{\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}\times\frac{6}{7}\times\frac{7}{8}\times\frac{8}{9}}{(0.1+0.9)+(0.2+0.8)+(0.3+0.7)+(0.4+0.6)+0.5}$$

$$=\frac{\frac{1}{9}}{\frac{9}{2}}=\frac{2}{81}.$$

20. Solution: $\frac{61}{3}$.

$1 - 2 \div 3 + 4 \times 5 = 20\frac{1}{3} = \frac{61}{3}$.

21. Solution: 8.

3	w	
x	y	4
7	z	

$3 + 7 = y + 4 \quad \Rightarrow \quad y = 6$

$4 + z = 2 \times 3 \quad \Rightarrow \quad z = 2$

$4 + w = 2 \times 7 \quad \Rightarrow \quad w = 10$

$z + w = x + 4 \quad \Rightarrow \quad 2 + 10 = x + 4 \quad \Rightarrow \quad x = 8.$

22. Solution: 3.

$(\frac{1}{10} + \frac{1}{15} + \frac{1}{20}) \times 3 + (\frac{1}{15} + \frac{1}{20}) \times x = 1 \quad \Rightarrow \quad x = 3.$

23. Solution: $\frac{16}{5}$.

Method 1:

$1 \div [(1 \div 1\frac{4}{5}) - \frac{35}{144}] = \frac{16}{5}$ hours.

Method 2:

Planet A completes $144 \div 1\frac{4}{5} = 80$ rounds.

Planet B completes $80 - 35 = 45$ rounds.

The time needed for Planet B to complete one round is $144 \div 45 = \frac{16}{5}$ hours.

24. Solution: $\frac{30}{31}$.

p and $p+1$ are consecutive integers. So $p = 2$ and $p+1 = 3$. Then $p+3 = 5$.

$1 \div (\frac{1}{2} + \frac{1}{3} + \frac{1}{5}) = \frac{30}{31}$

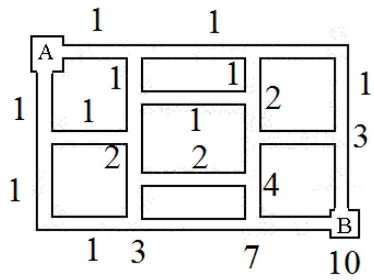
25. Solution: 13.

Let the original 2-digit positive integer be $\overline{ab}$ and the new number be $\overline{a0b}$.

So we have $8\times\overline{ab}=\overline{a0b}+1 \quad\Rightarrow\quad 8(10a+b)=100a+b+1 \quad\Rightarrow\quad 20a+1=7b$.

We know that a and b are all digits. So a = 1 and b = 3. $\overline{ab}$ = 13.

26. Solution: 10.



27. Solution: 903

From 10 : 00 to 10 : 30, the first 3 numbers are 1, 0, 2.

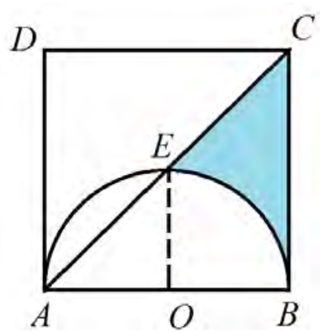
Let the time be 10 : 2*a* : *bc*. *b* can be 3, 5, or 4. *a* and *c* can be 3, 4, 5, 6, 7, 8, 9.

We first determine *b* with 3 ways, then determine *a* with 6 ways, and last we determine *c* with 5 ways.

The answer si 3 × 6 × 5 = 60.

28. Solution: $\frac{75}{2}-\frac{25}{4}\pi$

Draw $EO \perp AB$ at O. We see that $\angle CAB = 45°$. O is the center of the half circle.



The shaded area is $S_{EOBC}-S_{OEB}$

$= (5 + 10) \times 5 \div 2 - \frac{1}{4}\pi\times 5^2 = \frac{75}{2}-\frac{25}{4}\pi$

29. Solution: 8,000.

His salary in each month is [(12800 – 8000) + (24 × 800 – 18 × 1000)] = 1000.

The amount of money now in his moneybox IS 8000 – 18 × (1000 – 1000) = $8,000.

30. Solution: 10.

Let a be the amount of salt. After the first addition, the amount of salt becomes b grams.

$$\frac{a}{b} = \frac{15}{100} \qquad (1)$$

When c gram of water added, we have $\frac{a}{b+c} = \frac{12}{100}$ (2)

So we have $\frac{b}{a} = \frac{100}{11}$ and $\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a} = \frac{100}{12}$.

$$\frac{c}{a} = \frac{100}{12} - \frac{100}{15} = \frac{100}{60}$$

$$\frac{b+2c}{a} = \frac{b+c}{a} + \frac{c}{a} = \frac{100}{12} + \frac{100}{60} = \frac{100}{10}$$

After the same amount of water is added the third time, The salt content becomes:

$$\frac{a}{b+2c} = \frac{10}{100} = 10\%$$